Reasons:

Three Essays on their Logic, Pragmatics, and Semantics

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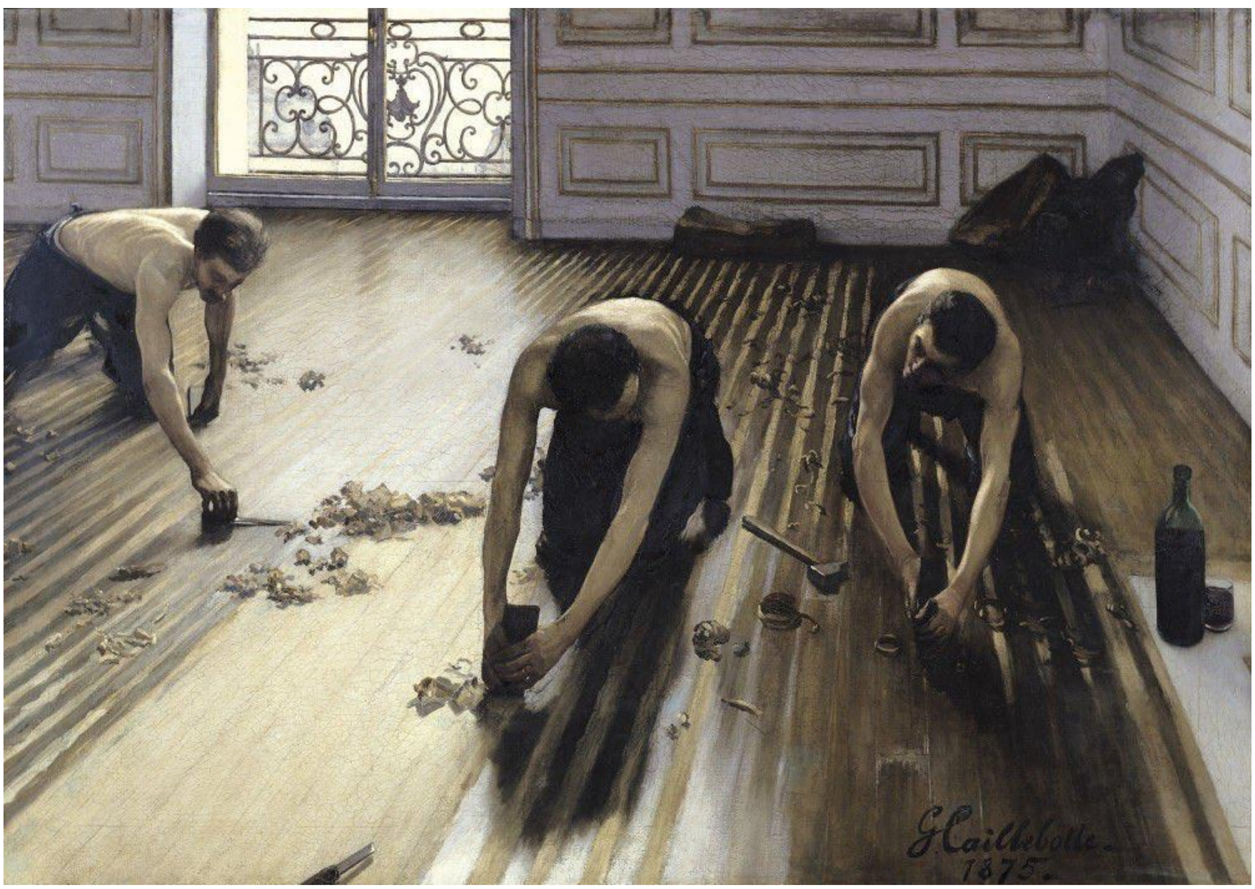

Reasons: Three Essays on their Logic, Pragmatics, and Semantics

FROM LOGICAL EXPRESSIVISM TO EXPRESSIVIST LOGIC: SKETCH OF A PROGRAM AND SOME IMPLEMENTATIONS[1]

Robert Brandom

University of Pittsburgh

I. Introduction

Traditionally, two principal issues in the philosophy of logic are the demarcation question (what distinguishes specifically *logical* vocabulary?) and the correctness question (what is the *right* logic?). One of the binding-agents tying together semantic and logical inferentialism is a distinctive philosophy of logic: logical expressivism. This is the view that the expressive role that distinguishes logical vocabulary is to make explicit the inferential relations that articulate the semantic contents of the concepts expressed by the use of ordinary, nonlogical vocabulary. If one offers this logically expressivist, semantically inferentialist answer to the demarcation question, the correctness question lapses.

It is replaced by a concrete task. For each bit of vocabulary to count as logical in the expressivist sense, one must say what feature of reasoning, to begin with, with *non*logical concepts, it expresses. Instead of asking what the *right* conditional is, we ask what dimension of normative assessment of implications various conditionals make explicit. For instance, the poor, benighted, and unloved, classical two-valued conditional makes explicit the sense of "good inference" in which it is a good thing if an inference does *not* have true premises and a false conclusion. (At least we can acknowledge that implications that do *not* have at least this property are bad.) Intuitionistic conditionals in the broadest sense let us assert that there is a procedure for turning an argument for the premises of an inference into an argument for the conclusion. C.I. Lewis's hook of strict implication codifies the sense in which it is a good

[1] The proof-theoretic logical systems I report on in this paper were developed as the result of many years of work in our logic working group at the University of Pittsburgh, brought to fruition by Ulf Hlobil and Dan Kaplan.

feature of an inference if it is *impossible* for its premises to be true and its conclusion not to be true. And so on. There can in principle be as many conditionals as there are dimensions along which we can endorse implications.

In spite of its irenic neutrality concerning the correctness question, one might hope that a new approach to the philosophy of logic such as logical expressivism would not only explain features of our *old* logics but ideally also lead to *new* developments in logic itself. I think this is in fact the case, and I want here to offer a sketch of how.

II. Prelogical Structure

I take it that the task of logic is to provide mathematical tools for articulating the *structure* of reasoning. Although for good and sufficient historical reasons, the original test-bench for such tools was the codification of specifically *mathematical* reasoning, the expressive target ought to be reasoning generally, including for instance and to begin with, its more institutionalized species, such as reasoning in the empirical sciences, in law-courts, and in medical diagnosis.

We can approach the target-notion of the <u>structure</u> of reasoning in two stages. The first stage distinguishes what I will call the "relational structure" that governs our reasoning practices. Lewis Carroll's fable "Achilles and the Tortoise" vividly teaches us to distinguish, in John Stuart Mill's terms, "premises from which to reason" (including those codifying implication relations) from "rules in accordance with which to reason," demonstrating that we cannot forego the latter wholly in favor of the former. Gil Harman sharpens the point in his argument that there is no such thing as rules of deductive reasoning. If there were, presumably a paradigmatic one would be: If you believe *p* and you believe *if p then q*, then you should believe *q*. But that would be a terrible rule. You might have much better reasons against *q* than you have for either of the premises. In that case, you should give up one of them. He concludes that we should distinguish *relations* of *implication*, from *activities* of *inferring*. The fact that *p, if p then q*, and *not*-*q* are incompatible, because *p* and *if p then q* stand in the implication relation to *q*, normatively *constrains* our reasoning activity, but does not by itself *determine* what it is correct or incorrect to do.

The normative center of reasoning is the practice of assessing reasons for and against conclusions. Reasons *for* conclusions are normatively governed by relations of *consequence* or *implication*. Reasons *against* conclusions are normatively governed by relations of *incompatibility*. These relations of implication and incompatibility, which constrain normative assessment of giving reasons for and against claims, amount to the first significant level of *structure* of the *practice* of giving reasons for and against claims.

These are, in the first instance, what Sellars called "*material*" relations of implication and incompatibility. That is, they do not depend on the presence of logical vocabulary or concepts, but only on the contents of non- or pre-logical concepts. According to semantic inferentialism, these are the relations that articulate the conceptual contents expressed by the *prelogical* vocabulary that plays an essential role in formulating the premises and conclusions of inferences.

Once we have distinguished these *relations* from the practice or activity of reasoning that they normatively govern, we can ask after the *algebraic* structure of such relations. In 1930s, Tarski and Gentzen, in the founding documents of the model-theoretic and proof-theoretic traditions in the semantics of logic, though differing in many ways in their approaches (as Jarda discusses in the second half of his book), completely agree about the algebraic structure of *logical* relations of consequence and incompatibility. Logical consequence satisfies Contexted Reflexivity (or Containment), Monotonicity, and Idempotence (Gentzen's "Cut", sometimes called "Cumulative Transitivity"). In Tarski's terms: $X \subseteq Cn(X)$, $X \subseteq Y \Rightarrow Cn(X) \subseteq Cn(Y)$, and $Cn(Cn(X))=Cn(X)$. Logical incompatibility satisfies what Peregrin calls "explosion": the implication of *everything* by logically inconsistent sets. (Peregrin builds this principle in so deeply that he takes the functional expressive role of negation to be serving as an "explosion detector.")

Perhaps these are, indeed, the right principles to require of specifically *logical* relations of consequence and incompatibility. But logical expressivists must ask a prior question: **What is**

the structure of *material* relations of consequence and incompatibility? This is a question the tradition has not thought about at all. But the answer one gives to it substantially shapes the logical enterprise when it is construed as expressivism does.

We can think of statements of implication and incompatibility as expressing what is *included in* a premise-set and what is *excluded by* it. In a semantic inferentialist spirit, we can say that the elements of a premise-set are its *explicit* content, and its consequences are its *implicit* content—in the literal sense of what is *implied by* it. It is reasonable to suppose that what is *explicitly* contained in a premise-set is also part of its *implicit* content. It is accordingly plausible to require that material consequence relations, no less than logical ones, be reflexive in an extended sense: if the premises explicitly contain a sentence, they also implicitly contain it, regardless of what other auxiliay premises are available. (We sometimes call this condition "Containment", thinking of Tarski's algebraic closure principle that every premise-set is a subset of its consequence-set.)

Monotonicity, by contrast, is *not* a plausible constraint on *material* consequence relations. It requires that if an implication (or incompatibility) holds, then it holds no matter what additional auxiliary hypotheses are added to the premise-set. But outside of mathematics, almost all our actual reasoning is *defeasible*. This is true in everyday reasoning by auto mechanics and on computer help lines, in courts of law, and in medical diagnosis. (Indeed, the defeasibility of medical diagnoses forms the basis of the plots of every episode of "House" you have ever seen—besides all those you haven't.) It is true of subjunctive reasoning generally. If were to I strike this dry, well-made match, it would light. But *not* if it is in a very strong magnetic field. Unless, additionally, it were in a Faraday cage, in which case it would light. But *not* if the room were evacuated of oxygen. And so on.

The idea of "laws of nature" reflects an approach to subjunctive reasoning deformed by a historically conditioned, Procrustean ideology whose shortcomings show up in the need for idealizations (criticized by Cartwright in *How the Laws of Physics Lie*) and for "physics avoidance" (diagnosed by Wilson in *Wandering Significance* on the basis of the need to invoke supposedly "higher-level" physical theories in *applying* more "fundamental" ones).

Defeasibility of inference, hence nonmonotonicity of implication relations, is a structural feature not just of probative or permissive reasoning, but also of dispositive, committive reasoning. *Ceteris paribus* clauses do not magically turn nonmonotonic implications into monotonic ones. (The proper term for a Latin phrase whose recitation can do *that* is "magic spell.") The expressive function characteristic of *ceteris paribus* clauses is rather explicitly to *mark* and *acknowledge* the defeasibility, hence nonmonotonicity, of an implication codified in a conditional, not to cure it by *fiat*.

The logical expressivist (including already—as I've argued elsewhere—Frege in the *Begriffsschrift*, at the dawn of modern logic) thinks of logical vocabulary as introduced to let one *say* in the logically extended object-language what material relations of implication and incompatibility articulate the conceptual contents of logically atomic expressions (and, as a bonus, to express the relations of implication and incompatibility that articulate the contents of the newly introduced logical expressions as well). There is no good reason to restrict the expressive ambitions with which we introduce logical vocabulary to making explicit the rare material relations of implication and incompatibility that are monotonic. Comfort with such impoverished ambition is a historical artifact of the contingent origins of modern logic in logicist and formalist programs aimed at codifying specifically *mathematical* reasoning. It is to be explained by appeal to historical causes, not good philosophical reasons.

Of course, since our tools were originally designed with this task in mind, as we have inherited them they are best suited for the expression of monotonic rational relations. But we should not emulate the drunk who looks for his lost keys under the lamp-post rather than where he actually dropped them, just because the light is better there. We should look to shine light where we need it most.

Notice that reasons *against* a claim are as defeasible in principle as reasons *for* a claim. Material incompatibility relations are no more monotonic in general than material implication relations. Claims that are incompatible in the presence of one set of auxiliary hypotheses can in some cases be reconciled by suitable additions of collateral premises. Cases with this shape are not hard to find in the history of science.

What about Cut, the principle of cumulative *transitivity*? It is expressed in Tarski's algebraic metalanguage for consequence relations by the requirement that the consequences of the consequences of a premise-set are just the consequences of that premise-set, and by Gentzen as the principle that adding to the explicit premises of a premise-set something that is already part of its implicit content does not add to what is implied by that premise-set.

Thought of this way, Cut is the dual of what is usually thought of as the weakest acceptable structural principle that must be required if full monotonicity is not.[2] "Cautious monotonicity" is the structural requirement that adding to the explicit content of a premise-set sentences that are already part of its implicit content not defeat any implications of that premise-set. (Even though there might be *some* additional premises that *would* infirm the implication, sentences that are *already implied* by the premise-set are not among them.)

We can think generally about the structural consequences of the process of *explicitation* of content, in the sense of making what is *implicitly* contained in (or excluded by) a premise-set, in the sense of being implied by it, *explicit* as part of the explicit premises. Cut says that explicitation never *adds* implicit content. Cautious monotonicity says that explicitation never *subtracts* implicit content. Together they require that ***explicitation* is *inconsequential***. Moving a sentence from the right-hand side of the implication-turnstile to the left-hand side does not change the consequences of the premise-set. It has no effect whatever on the implicit content, on what is implied. (Explicitation can also involve making explicit what is implicitly *excluded* by a premise-set.)

Explicitation in this sense is not at all a *psychological* matter. And it is not even yet a strictly *logical* notion. For even *before* logical vocabulary has been introduced, we can make

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[2] On holding onto both Cut and Cautious Monotonicity, see Gabbay, D. M., 1985, "Theoretical foundations for nonmonotonic reasoning in expert systems", in K. Apt (ed.), *Logics and Models of Concurrent Systems*, Berlin and New York: Springer Verlag, pp. 439–459. Gabbay agrees with the criteria of adequacy laid down by the influential KLM approach of Kraus, Lehman, and Magidor: Kraus, Sarit, Lehmann, Daniel, & Magidor, Menachem, 1990. Nonmonotonic Reasoning, Preferential Models and Cumulative Logics. *Artifical Intelligence*, 44: 167–207.

sense of explicitation in terms of the structure of *material* consequence relations. Noting the effects on implicit content of adding as an explicit premise sentences that were already implied is already a process available for investigation at the semantic level of the *prelogic*.

It might well be sensible to require the inconsequentiality of explicitation as a structural constraint on *logical* consequence relations. But just as for the logical expressivist there is no good reason to restrict the rational relations of implication and incompatibility we seek to express with logical vocabulary to monotonic ones, there is no good reason to restrict our expressive ambitions to consequence relations for which explicitation is inconsequential. On the contrary, there is every reason to want to use the expressive tools of logical vocabulary to investigate cases where explicitation *does* make a difference to what is implied.

One such case of general interest is where the explicit contents of a premise-set are the records in a *database*, whose implicit contents consist of whatever consequences can be extracted from those records by applying an *inference engine* to them. (The fact that the "sentences" in the database whose material consequences are extracted by the inference engine are construed to begin with as *logically* atomic does not preclude the records having the "internal" structure of the arbitrarily complex datatypes manipulated by any object-oriented programming language.) It is by no means obvious that one is obliged to treat the results of applying the inference-engine as having exactly the same epistemic status as actual entries in the database. A related case is where the elements of the premise-sets consist of experimental *data*, perhaps measurements, or observations, whose implicit content consists of the consequences that can be extracted from them by applying a *theory*. In such a case, explicitation is far from inconsequential. On the contrary, when the CERN supercollider produces observational measurements that confirm what hitherto had been purely theoretical predictions extracted from previous data, the transformation of rational status from *mere* prediction *implicit* in prior data to actual empirical observation is an event of the first significance—no less important than the observation of something incompatible with the predictions extracted by theory from prior data. This is the very nature of empirical *confirmation* of theories. And it often happens that confirming *some* conclusions extracted by theory from the data infirms *other* conclusions that one otherwise would have drawn.

Imposing Cut and Cautious Monotonicity as global structural constraints on material consequence relations amounts to equating the epistemic status of premises and conclusions of good implications. But in many cases, we want to acknowledge a distinction, assigning a lesser status to the products of risky, defeasible inference. In an ideal case, perhaps this distinction shrinks to nothing. But we also want to be able to reason in situations where it is important to keep track of the difference in status between what we take ourselves to know and the shakier products of our theoretical reasoning from those premises. We shouldn't build into our global structural conditions on admissible material relations of implication and incompatibility assumptions that preclude us from introducing logical vocabulary to let us talk about those rational relations, so important for confirmation in empirical science.

The methodological advice not unduly to limit the structure of rational relations to which the expressive ambitions of our logics extend applies particularly forcefully to the case of incompatibility relations. The structural constraint the classical tradition for which Gentzen and Tarski speak imposes on incompatibility relations is *explosion*: the requirement that from incompatible premises anything and everything follows. This structural constraint corresponds to nothing whatsoever in ordinary reasoning practices, not even as institutionally codified in legal or scientific argumentative practices. It is a pure artifact of classical logical machinery, the opportune but misleading translation of the two-valued conditional into a constraint on implication and incompatibility that reflects no corresponding feature of the practices that apparatus—according to the logical expressivist—has the job of helping us to talk about. It is for that reason a perennial embarrassment to teachers of introductory logic, who are forced on this topic to adopt the low invocations of authority, pressure tactics, and rhetorical devices otherwise associated with commercial hucksters, con men, televangelists, and all the other sophists from whom since Plato we have hoped to distinguish those who are sensitive to the normative force of the better reason, whose best practices, we have since Aristotle hoped to codify with the help of logical vocabulary and its rules. In the real world, we are often obliged to reason from sets of premises that are explicitly or implicitly incompatible. [An extreme case is the legal practice of "pleading in the alternative." My defense is first, that I never borrowed the lawnmower, second, that it was broken when you lent it to me, and third that it was in perfect condition when I returned it. You have to show that *none* of these things is true. In pleading this way I am not confessing to having assassinated Kennedy. Examples from high scientific theory are not far to seek.] We should not impose structural conditions in our *prelogic* that preclude us from *logically* expressing material relations of incompatibility that characterize our actual reasoning. Explosion is not a plausible structural constraint on material relations of incompatibility, and our logic should not require us to assume that it is.

III. The Expressive Role of Basic Logical Vocabulary.

The basic claim of logical expressivism in the philosophy of logic is that the expressive role characteristic of *logical* vocabulary is to make explicit, in the object-language, relations of implication and incompatibility, including the material, prelogical ones that, according to semantic inferentialism, articulate the conceptual contents expressed by nonlogical vocabulary, paradigmatically ordinary empirical descriptive vocabulary. The paradigms of logical vocabulary are the *conditional*, which codifies relations of implication that normatively structure giving reasons *for* claims, and *negation*, which codifies relations of incompatibility that normatively structure giving reasons *against* claims.

To say that a premise-set implies a conclusion, we can write in the metalanguage: "$\Gamma|\!\sim A$". To say that a premise-set is incompatible with a conclusion, we can write in the metalanguage "$\Gamma, A|\!\sim \bot$".

To perform its defining expressive task of codifying implication relations in the object language, conditionals need to satisfy the

Ramsey Condition: $\mathbf{\Gamma|\!\sim A \rightarrow B \;\; iff \;\; \Gamma, A|\!\sim B.}$

That is, a premise-set implies a conditional just in case the result of adding the antecedent to that premise-set implies the consequent. A conditional that satisfies this equivalence can be called a "Ramsey-test conditional," since Frank Ramsey first proposed thinking of conditionals this way.

To perform its expressive task of codifying incompatibility relations in the object language, negation needs to satisfy the

Minimal Negation Condition: $\mathbf{\Gamma|\!\sim\neg A}$ **iff** $\mathbf{\Gamma,A|\!\sim\bot}$**.**

That is, a premise-set implies not-A just in case A is incompatible with that premise-set. (It follows that $\neg A$ is the minimal incompatible of A, in the sense of being implied by everything that is incompatible with A.)

We should aspire to expressive logics built onto material incompatibility relations that are nonmonotonic as well as material implication relations that are nonmonotonic. That means that just as an implication $\Gamma|\!\sim A$ can be defeated by adding premises to Γ, so can an incompatibility. Sometimes, $\Gamma,A|\!\sim\bot$ can also be defeated, the incompatibility "cured", by adding some additional auxiliary hypotheses to Γ. And while, given the role negation plays in codifying incompatibilities, an incompatible set, $\Gamma\cup\{A\}$ that is, one such that $\Gamma,A|\!\sim\bot$) will imply the negations of all the premises that are its explicit members, it need not therefore imply *everything*. In substructural expressive logics built on Gentzen's multisuccedent system, the condition that emerges naturally is not *ex falso quodlibet*, the classical principle of explosion, but what Ulf Hlobil calls "*ex fixo falso quodlibet*" (EFF). This is the principle that if Γ is not only materially incoherent (in the sense of explicitly containing incompatible premises) but *persistently* so, that is *incurably*, *indefeasibly* incoherent, in that *all* of its supersets are also incoherent, *then* it implies everything. In a monotonic setting, this is equivalent to the usual explosion principle. In nonmonotonic settings, the two conditions come apart. One conclusion that might be drawn from expressive logics is that what mattered all along was always *ex fixo falso quodlibet*—classical logic just didn't have the expressive resources to distinguish this from explosion of all incoherent sets.

The basic idea of expressivist logic is to start with a language consisting of nonlogical (logically atomic) sentences, structured by relations of material implication and incompability. In the most general case, we think of those relations as satisfying the structural principles *only* of extended reflexivity—not monotonicity, not cautious monotonicity, and not even transitivity in the form of Cut. We then want to introduce logical vocabulary on top of such a language. This means extending the language to include arbitrarily logically complex sentences formed from the logically atomic sentences by repeatedly applying conditionals and negations, and then extending the underlying material consequence and incompatibility relations to that logically extended language in such a way that the Ramsey condition and the Minimal Negation Condition both hold. (In fact, we'll throw in conjunction and disjunction as well.)

A basic constraint on such a construction is set out by a simple argument due to Ulf Hlobil.[3] He realized that in the context of Contexted Reflexivity and a Ramsey conditional, Cut entails Monotonicity. For if we start with some arbitrary implication $\Gamma|\sim A$, we can derive $\Gamma,B|\sim A$ for arbitrary B—that is, we can show that arbitrary additions to the premise-set, arbitrary weakenings of the implication, preserves those implications. And that is just monotonicity. For we can argue:

$\underline{\Gamma\|\sim A}$	Assumption
$\underline{\Gamma,A, B\|\sim A}$	Contexted Reflexivity
$\underline{\Gamma,A\|\sim B\rightarrow A}$	Ramsey Condition Right-to-Left
$\underline{\Gamma\|\sim B\rightarrow A}$	Cut, Cutting A using Assumption
$\Gamma,B\|\sim A$	Ramsey Condition Left-to-Right.

Since we want to explore adding Ramsey conditionals to codify material implication relations that are reflexive but do not satisfy Cut—so that prelogical explicitation is not treated as always inconsequential—we will sacrifice Cut in the logical extension.

It is a minimal condition of logical vocabulary playing its defining expressive role that introducing it extend the underlying material consequence and incompatibility relations *conservatively*. (Belnap motivates this constraint independently, based on considerations raised by Prior's toxic "tonk" connective. The logical expressivist has independent reasons to insist on conservativeness: only vocabulary that conservatively extends the material relations of consequence and incompatibility on which it is based can count as *expressing* such relations explicitly.) So there should be no implications or incompatibilities involving only *old* (nonlogical) vocabulary that hold or fail to hold in the structure logically extended to include *new*, logical vocabulary, that do not hold or fail to hold already in the material base structure.

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[3] Hlobil, U. (2016), "A Nonmonotonic Sequent Calculus for Inferentialist Expressivists." In Pavel Arazim and Michal Dančák (eds.) The *Logica* Yearbook 2015, pp. 87-105, College Publications: London.

Since that material base structure is in general nonmonotonic and intransitive, satisfying only contexted reflexivity, so must be the global relations of consequence and incompatibility that result from extending them by adding logical vocabulary.

IV. Basic Expressivist Logics

We now know how to do that in the context of Gentzen-style substructural proof theory. I will be summarizing technical work by recent Pitt Ph.D. Ulf Hlobil, now at Concordia University (on single-succedent systems) and current Pitt Ph.D. student Dan Kaplan (on multi-succedent systems).

We produce substructural logics codifying consequence and incompatibility relations that are not globally monotonic or transitive by modifying Gentzen's systems in three sequential stages. Gentzen's derivations all begin with what he called "initial sequents," in effect, axioms, (which will be the leaves of all *logical* proof trees) that are instances of immediate or simple reflexivity. That is, they are all of the form A|~A. We impose instead a structural rule that adds all sequents that are instances of *contexted* reflexivity—that is, (in the multisuccedent case) all sequents of the form Γ,A|~A,Θ. Making this change does not really change Gentzen's system LK of classical logic at all. For he can derive the contexted version from immediate Reflexivity by applying Monotonicity, that is Weakening (his "Thinning"). So, as others have remarked, Gentzen does not need the stronger principle of unrestricted monotonicity in order to get the full system LK of classical logic. He can make do just with the *very* restricted monotonicity principle of Contexted Reflexivity, which allows arbitrary weakening *only* of sequents that are instances of reflexivity, that is, which have some sentence that already appears on both sides of the sequent one is weakening. Since all Gentzen's initial sequents are instances of immediate reflexivity, being able to weaken them turns out to be equivalent to being able to weaken *all* logically derivable sequents. (The weakenings can be "permuted up" the proof trees past applications of connective rules in very much the same way Gentzen appeals to in proving his

Cut-Elimination Hauptsatz.) Substituting the stronger version of Reflexivity for Gentzen's version accordingly allows dropping the structural requirement of Monotonicity. Contexted Reflexivity arises most naturally in Tarski's algebraic-topological way of thinking about consequence relations, as the principle that each premise-set is contained in its consequence set: $\Gamma \subseteq \mathrm{con}(\Gamma)$.

We also do not impose Cut as a global structural constraint. But Gentzen's Cut-Elimination theorem will still be provable for all proof-trees whose leaves are instances of (now, contexted) Reflexivity. So the purely logical part of the system will still satisfy Cut.

The next step in modifying Gentzen's systems is to add axioms in the form of initial sequents relating logically atomic sentences that codify the initial base of *material* implications (and incompatibilities). Whenever some premise-set of atomic sentences Γ_0 implies an atomic sentence A, we add $\Gamma_0 |\!\sim A$ to the initial sequents that are eligible to serve as leaves of proof-trees, initiating derivations. (We require that this set of sequents, too, satisfies Contexted Reflexivity. We will be able to show that the connective rules preserve this property.) This is exactly the way Gentzen envisaged substantive axioms being added to his logical systems so that those systems could be used to codify substantive theories—for instance, when he considers the consistency of arithmetic. The crucial difference is that he required that these sequents, like those governing logically complex formulae, satisfy the structural conditions of Monotonicity and Cut—and we do not. We will introduce logical vocabulary to extend material consequence and incompatibility relations that do *not* satisfy Monotonicity, and that are *not* idempotent.

The third stage in modifying Gentzen's systems is accordingly to extend the pre-logical language to include arbitrarily logically complex sentences formed from that pre-logical vocabulary by the introduction of logical connectives. Gentzen's connective rules show how antecedent consequence and incompatibility relations governing the logically atomic base language can be systematically extended so as to govern the sentences of the logically extended language. Gentzen's own rules can be used to do this, with only minor tweaks. Like Ketonen's version of Gentzen's rules, ours are *reversible*. They are unlike the Gentzen-Ketonen rules in that we mix additive and multiplicative rules. They are all equivalent to Gentzen's own rules in the presence of a global structural rule of Monotonicity. But in nonmonotonic settings, they come apart. So, for instance, Gentzen's left rule for conjunction allows us to move from Γ,A|~C to Γ,A&B|~C. That builds in monotonicity on the left. We can't have that, since in the material base, it can happen that adding B as a further premise defeats the implication of C by Γ and A. We allow instead only the move from Γ,A,B|~C to Γ,A&B|~C. (A similar shift is needed in his right rule for disjunction: where he allows derivation of Γ|~AvB,Θ from Γ|~A,Θ, building in monotonicity on the right, we allow instead only the move from Γ|~A,B,Θ to Γ|~AvB,Θ.)

I said above that from a logical expressivist point of view, for the conditional to do its defining job of codifying implication relations in the object language, it needs to satisfy the Ramsey condition. In Gentzen's setting, this amounts to the two principles:

$$\text{CP:}\quad \frac{\Gamma,A|\!\sim B}{\Gamma|\!\sim A\rightarrow B} \qquad \text{and} \qquad \text{CCP:}\quad \frac{\Gamma|\!\sim A\rightarrow B}{\Gamma,A|\!\sim B.}$$

The first is Gentzen's right-rule for the conditional. The second rule is not one of his. And it cannot be. For it is a simplifying rule. The only simplifying rule he has is Cut, and it is of the essence of his program to show that he can do without that rule: that every derivation that appeals to that single simplifying rule can be replaced by a derivation that does not appeal to it. Ketonen-style invertibility of connective rules, which makes root-first proof searches possible, though, requires not only Conditional Proof but the simplifying rule Converse Conditional Proof. And it is possible to show that this rule, too, like Cut is "admissible" in Gentzen's sense: every derivation that uses it can be replaced by a derivation that does not.

It can be shown that our versions of Gentzen's connective rules produce a *conservative extension* of any *nonmonotonic* material base consequence relation (including nonmonotonic incompatibility relations incorporated in such consequence relations) that satisfies the structural condition of Contexted Reflexivity. That is, in the absence of explicitly imposing a structural rule of Monotonicity (Weakening or Thinning) and Cut, the connective rules do not force global monotonicity. So the resulting, logically extended consequence relation is nonmonotonic. And the nonmonotonicity extends to logically complex formulae, for instance, as we have seen, in that from the fact that Γ,A|~C it does not follow that Γ,A&B|~C, so that from Γ|~A→C it does not follow that Γ|~(A&B)→C. The logical language that results permits the explicit codification using ordinary logical vocabulary of arbitrary *nonmonotonic*, insensitive material consequence relations in which prelogical explicitation is *not* inconsequential.

And yet, the system is supraclassical. All the theorems of Gentzen's system LK of classical logic can be derived in this system. For if we restrict ourselves to derivations all of whose leaves are instances of Contexted Reflexivity, that is, are of the form Γ,A|~A,Θ, the result is just the theorems of classical logic. It is only if we help ourselves to initial sequents that are *not* of that form, the axioms that codify *material* relations of consequence and incompatibility, that we derive nonclassical results. **Gentzen never needed to require monotonicity, his "Thinning," as a *global* structural rule. He could just have used initial sequents that correspond to Contexted Reflexivity instead of immediate reflexivity**. That gives him all the weakening behavior he needs. Further, if we look only at sequents that are derivable *no matter what material base relation we extend,* sequents such as Γ,A,A→B|~B, hence Γ|~(A&(A→B))→B, we find that the "logic" of our system in this sense, too, is just classical logic. Perhaps not surprisingly, if, following Gentzen, we use essentially the same connective rules but restrict ourselves to *single* succedent sequents, the result is a globally nonmonotonic, intransitive *supraintuitionist* logic.[4]

I have been talking about the logical extension of nonmonotonic material *consequence* relations and not about the logical extension of nonmonotonic material *incompatibility* relations. But the latter are equally well-behaved. The multi-succedent connective rules for negation are just Gentzen's. But it is *not* the case that any materially incoherent premise-set implies every sentence. Such premise-sets imply both the sentences they explicitly contain and the negations of all those sentences. But they do not imply everything else. If a premise-set explicitly contains both A and ¬A for some sentence A, *then* it implies everything. But that is because *persistently* or monotonically incoherent premise-sets explode—that is, sets that are not only incoherent themselves, but such that *every superset* of them is incoherent. This is what Ulf Hlobil calls "*ex **fixo** falso quodlibet.*" No specific stipulation to this effect needs to be made. It arises naturally out of the connective rules in the multisuccedent setting. If monotonicity held globally, *ex falso*

[4] We do have to add some special rules, to make up for some of the things that happen on the right in the cleaner multisuccedent system.

quodlibet and *ex fixo falso quodlibet*" would be equivalent. Outside of derivations all of whose leaves are instances of contexted reflexivity, in our systems, they are not.

So in a clear sense, the *logic* is monotonic and transitive—indeed, classical or intuitionistic, depending (as with Gentzen) on whether we look at multi-succedent or single-succedent formulations—but the logically extended consequence and incompatibility relations *in general*, are not.[5] The logic of nonmonotonic consequence relations is itself monotonic. Yet it can *express*, in the logically extended object language, the nonmonotonic relations of implication and incompatibility that structure both the material, prelogical base language, and the logically compound sentences formed from them, as they behave in derivations that substantially depend on the material base relations.

Substructural expressivist logics suitable for making explicit nonmonotonic, nontransitive material consequence and incompatibility relations are accordingly not far to seek. They can easily be built by applying to nonlogical axioms codifying those material relations of implication and incompatibility variants of Gentzen's connective definitions that are equivalent to his under his stronger structural assumptions. It turns out that the relations of implication and incompatibility that hold in virtue of their logical form alone are still monotonic and transitive, even though the full consequence and implication relations codified by the logical connectives is not. So if you want Cut and Weakening, you can still have them—for purely *logical* consequence. Remember that from the point of view of logical expressivism, the point of introducing logical vocabulary is not what you can *prove* with it (what implications and incompatibilities hold in virtue of their logical form alone) but what you can *say* with it. Expressivist logics let us *say* a *lot* more than is said by their logical theorems.

V. Codifying *Local* Regions of Structure: Monotonicity as a Modality

[5] When I talk about "the logic" here this can mean *either* the theorems derivable just from instances of Contexted Reflexivity (following Gentzen) or what is implied by every premise-set for every material base relation of implication and incompatibility that satisfies Contexted Reflexivity.

The master-idea of logical expressivism is that logical vocabulary and the concepts such vocabulary expresses are distinguished by playing a characteristic expressive role. They let us talk, in a logically extended object language, about the *material* relations of implication and incompatibility—what is a reason for and against what—that already articulate the conceptual contents of *non*logical vocabulary, as well as the *logical* relations of implication and incompatibility built on top of those material relations. Expressivist logics are motivated by the idea that we unduly restrict the expressive power of our logics if we assume that the global structural principles that have traditionally been taken to govern purely *logical* relations of consequence and inconsistency must be taken also to govern the underlying *material* consequence and incompatibility relations. So we don't presuppose Procrustean *global* structural requirements on the material relations of consequence and incompatibility we want to codify logically. Here is a further idea we have developed in what I am calling "expressivist logics." Instead of imposing structural constraints globally, we relax those conditions and introduce vocabulary that will let us *say explicitly*, in the logically extended object language, *that* they hold *locally*, wherever in fact they still do.

Material consequence relations, I have claimed, are not in general monotonic. But they are not always and everywhere *non*monotonic, either. *Some* material implications are *persistent*, in that they continue to hold upon arbitrary additions to their premises. It follows from the fact that the regular Euclidean planar polygon has more than three sides that its angles add up to more than 180º, no matter what additional premises we throw in. The mistake of the tradition was not to think that *there are* material implications like this, but to think that *all* material implications *must* be like this. Logical expressivists want to introduce logical vocabulary that explicitly marks the difference between those implications and incompatibilities that are persistent under the addition of arbitrary auxiliary hypotheses or collateral commitments, and those that are not. Such vocabulary lets us draw explicit boundaries around the islands of monotonicity to be found surrounded by the sea of nonmonotonic material consequences and incompatibilities.

From a Gentzenian perspective, expressivist logics work out a different way of conceiving the relations between *structure* and *connective rules*. Connectives are introduced to express local structures. The paradigm is the conditional, which codifies the implication turnstile, by satisfying the Ramsey condition in the form of CP and CCP. Conjunction codifies the comma on the left of the turnstile, and disjunction codifies the comma on the right of the turnstile (in multi-succedent systems). (Note that in our nonmonotonic setting, this requires multiplicative rather than additive rules for conjunction on the left and disjunction on the right.[6]) Negation codifies incompatibility (in Gentzen's multisuccedent systems elegantly captured in the *relation* between commas on the left and commas on the right). Our expressivist logics show how, in addition to the structures already captured by traditional connectives, further connectives can be introduced to mark local regions of the consequence relation where structure such as monotonicity and transitivity hold. I'll try to give some idea of how this works by sketching what is for us the paradigm case: monotonicity.

The first idea is to extend the expressive power of our proof-theoretic metalanguage, so as to be able to distinguish persistent implications. In addition to the generally nonmonotonic snake turnstile "$|\!\sim$", we can introduce a variant with an upward arrow, "$|\!\sim^{\uparrow}$" to mark persistent implications, that is, those that hold monotonically. To do this is to add *quantificational* expressive power to our proof-theoretic metalanguage. $\Gamma|\!\sim^{\uparrow}A$ says that not only does Γ imply A, but so does every superset of Γ: $\Gamma|\!\sim^{\uparrow}A$ iff $\forall X\subseteq L[\Gamma,X|\!\sim A]$.

All the connective rules can then be stipulated to have two forms: one for each turnstile. So we can write the right-rule (CP) for our Ramsey-test conditional showing the persistence arrow as optional, as:

$$\frac{\Gamma,A|\!\sim^{(\uparrow)}B}{\Gamma|\!\sim^{(\uparrow)}A\rightarrow B.}$$

If there is no upward arrow on the top turnstile, then there is none on the bottom either. But if there is a persistence-indicating upward arrow on the premise-sequent, then there is one also on the conclusion sequent. If Γ together with A *persistently* implies B—no matter what further premises we adjoin to them—then Γ *persistently* implies the conditional—no matter what further

6 $\frac{\Gamma,A,B|\!\sim\Theta}{\Gamma,A\&B|\!\sim\Theta}$ and $\frac{\Gamma|\!\sim A,B,\Theta}{\Gamma|\!\sim A\vee B,\Theta}$ rather than $\frac{\Gamma,A|\!\sim\Theta}{\Gamma,A\&B|\!\sim\Theta}$ $\frac{\Gamma,B|\!\sim\Theta}{\Gamma,A\&B|\!\sim\Theta}$ and $\frac{\Gamma|\!\sim A,\Theta}{\Gamma|\!\sim A\vee B,\Theta}$ $\frac{\Gamma|\!\sim B,\Theta}{\Gamma|\!\sim A\vee B,\Theta}$.

premises we adjoin to it. That follows from the original rule, together with the definition of persistence.

From this more structurally relaxed, nonmonotonic vantage point, traditional monotonic logic looks just the way it would if there were a notationally suppressed upward arrow on all of its turnstiles.

Incompatibility (and so logical inconsistency) also looks different in this setting. We now can distinguish materially incoherent premise-sets, where $\Gamma|\!\sim\bot$, from *persistently* incoherent premise-sets. These are premise-sets that are not only incoherent, but whose incoherence cannot be cured by the addition of further premises. And we can restrict explosion to those *persistently* incoherent sets. If $\Gamma|\!\sim\bot$, then for any $A\in\Gamma$, $\Gamma|\!\sim A$ and $\Gamma|\!\sim\neg A$. But it need *not* follow that for arbitrary B, $\Gamma|\!\sim B$. That follows only if $\Gamma|\!\sim^{\uparrow}\bot$. In the single-succedent case, we stipulate this: not *ex falso quodlibet* but *ex* ***fixo*** *falso quodlibet*: ExFF. In the multi-succedent case, we do not need this stipulation. It falls out of the standard Gentzen treatment of negation. Here we want to say that what was always right about the idea that everything follows from a contradiction (and in our systems, if $A\in\Gamma$ and $\neg A\in\Gamma$, then Γ is *persistently* incoherent, and *does* imply everything) is that *persistently* incoherent premise-sets imply everything. It's just that in rigidly monotonic systems, *all* incoherence is treated as persistent, so in that expressively impoverished setting, ExF and ExFF are equivalent.

Once the dual-turnstile apparatus is in place in the metalanguage, we can introduce a modal operator in the object language to let us say there *that* an implication holds persistently. The basic idea is to introduce a monotonicity-box that says that $\Gamma|\!\sim\Box A$ iff $\Gamma|\!\sim^{\uparrow}A$, that is, if and only if $\forall X\subseteq L[\Gamma,X|\!\sim A]$. To say that Γ implies $\Box A$ is just to say that Γ *persistently* (that is, monotonically) implies A. The monotonicity box is clearly a strong modality, in that if Γ implies $\Box A$, then it implies A. And it is an S4 modality, in that if Γ implies $\Box A$, then it implies $\Box\Box A$.

From the point of view of a globally nonmonotonic implication relation in which local pockets of monotonicity are marked in the object language by implication of modally qualified claims, the assumption of global monotonicity appears as what happens when one looks only at the monotonicity-*necessitations* of claims, ignoring anything not of the form $\Box A$.

In fact, we can do a lot better than what I have indicated so far. The expressivist idea is that the point of introducing logical vocabulary is to provide expressive resources that let one make explicit crucial local structural features of relations of implication and incompatibility—in the first instance, *material* relations of implication and incompatibility, and only as a sort of bonus the *logical* relations of implication and incompatibility that are built on top of them. From this point of view, what matters most is local persistence of some *material* implications. For it is these regions of local monotonicity in the material base relations of consequence and incompatibility that we want to be able to capture with a monotonicity-modal operator. Happily, it turns out that all we really need is an upward-arrow turnstile marking implications that can be weakened by the addition of arbitrary sets of logically *atomic* sentences. Our versions of Gentzen's connective rules then guarantee that arbitrary weakenings by sets of logically complex formulae will be possible when and only when arbitrary weakening by sets of atoms is possible according to the underlying material base consequence relation.

In addition to implications whose persistence is underwritten by peculiarities of the underlying material consequence relation, there are implications of sentences prefaced by the monotonicity box that reflect logical relations induced by the connective definitions. Sentences like these—for instance, $\Box(A \rightarrow A)$—do not depend on vagaries of the material implication relations.

A further innovation, pioneered by Ulf Hlobil for supra-intuitionistic single-succedent systems and by Dan Kaplan for supra-classical multiple-succedent systems, is the introduction of a much more powerful way of marking quantificational facts about sequents in the proof-theoretic metalanguage. (For simplicity, I'll continue to use the single-succedent case.) Instead of introducing a simple upward arrow, as I have appealed to in my sketch, we introduce an upward arrow subscripted with a set of sets. $\Gamma|\!\sim^{\uparrow X} A$ is defined as holding just in case for every set of sentences $X_i \in X$, $\Gamma, X_i |\!\sim A$. (In fact it suffices here, too, to restrict the values of X to sets of sets of logical *atoms* in the nonlogical material base language, but I put that complication aside here.) Then the set X specifies a set of sets of sentences that one can weaken Γ with, while preserving the implication of A. That is, it marks a *range of subjunctive robustness* of the implication Γ|~A. These are sets of sentences that can be added to Γ as collateral premises or auxiliary hypotheses without defeating the implication of A.

The underlying thought is that the most important information about a material implication is not whether or not it is monotonic—though that is something we indeed might want to know. It is rather under what circumstances it is robust and under what collateral circumstances it would be defeated. All implications are robust under *some* weakenings, and most are *not* robust under *all* weakenings. The space of material implications that articulates the contents of the nonlogical concepts those implications essentially depend upon has an intricate localized structure of subjunctive robustness and defeasibility. That is the structure we want our logical expressive tools to help us characterize. It is obscured by commitment to global structural monotonicity—however appropriate such a commitment might be for purely *logical* relations of implication and incompatibility.

Here, too, our variants of Gentzen's connective definitions, as well as those for the monotonicity box, are so contrived as to ensure that it suffices to look at ranges of subjunctive robustness of implications that are restricted to the logical atoms governed by *material* relations of consequence and incompatibility. The more fine-grained control over ranges of subjunctive robustness offered by the explicitly quantified upward arrow apparatus is governed by a couple of structural principles. To indicate their flavor: one lets us combine sets of sets under which a particular implication is robust:

$$\frac{\Gamma|\!\sim^{\uparrow X}A \qquad \Gamma|\!\sim^{\uparrow Y}A}{\Gamma|\!\sim^{\uparrow X\cup Y}A} \quad \text{Union}$$

If the implication of A by Γ is robust under weakening by all the sets in X and it is robust under weakening by all the sets in Y, then it is robust under weakening by all the sets in X∪Y. The very same connective rules stated with ordinary turnstiles go through as well with these quantified upward arrows with the same subjunctive-robustness subscript, and so propagate down proof trees.

The result of the addition of this apparatus is extensions of material consequence and incompatibility relations to a language including logically complex sentences, including those formed using the monotonicity modal box, that is well-defined and conservative of the material base relations. It follows that if the base relations are nonmonotonic and do not satisfy any version of Cut, then neither will the extended ones. The only structural principle we do impose on the base consequence relation, Contexted Reflexivity, is preserved. We do not impose the simplifying rule of Converse Conditional Proof (CCP)

$$\frac{\Gamma |\!\sim A \rightarrow B}{\Gamma, A |\!\sim B}$$

as a rule, but can prove it admissible, that is, as holding as a consequence of the connective rules for the conditional we do impose. The system is supraintuitionistic, in the single-succedent case, and supraclassical, in the multisuccedent case. If we restrict ourselves to elaborating material base consequence relations that consist entirely of instances of contexted reflexivity, that is of sequents of the form $\Gamma_0, p |\!\sim p$ for atomic sentences, then the logics over the extended languages are simply intuitionism and classical logic, respectively. These are obviously monotonic (so the monotonicity box is otiose), and Cut is, as usual, provably admissible.

VI. Conclusion

Construed narrowly, logical expressivism is a response to the demarcation question in the philosophy of logic. It suggests that we think of logical vocabulary and the concepts such vocabulary expresses as distinguished by playing a particular expressive role. The expressive task distinctive of logical vocabulary as such is to make explicit relations of consequence and incompatibility—to allow us to *say* what claims follows from other claims, and what claims rule out which others. Construed more broadly, logical expressivism invites us not to think about logic as having any autonomous subject matter—not *logical truth*, nor even *logical consequence*. Logic does not supply a canon of right reasoning, nor a standard of rationality. Rather, logic takes its place in the context of an already up-and-running rational enterprise of making claims and giving reasons for and against claims. Logic provides a distinctive *organ of self-consciousness* for such a rational practice. It provides expressive tools for talking and thinking, making claims, about the relations of implication and incompatibility that structure the giving of reasons for and against claims.

We should want those tools to be as broadly applicable as possible. The rational relations of material consequence that articulate the contents of nonlogical concepts are not in general monotonic. Good inferences can be infirmed be adding new information. Indeed, offering finitely statable reasons typically *requires* that the implications we invoke be defeasible. Logic should not ignore this fact, nor even aim to rectify it. Logic should aim rather to codify even nonmonotonic, intransitive reasoning.

What I have here called "expressive logics" do that. The tweaks required to the proof-theoretic apparatus Gentzen bequeathed us for it to be capable of codifying nonmonotonic, even intransitive, reasoning are remarkably small. That fact tends to confirm the expressivist's philosophical claims about what the *point* of logic has been all along. Expressive logics move beyond traditional logic not only in being built on antecedent relations of material consequence and incompatibility and in refusing to impose all but the most minimal global structural restrictions on those relations.[7] They also introduce logical vocabulary that lets one express, in the logically extended language with its logically extended relations of consequence and incompatibility, *local* regions where structural conditions *do* hold. The paradigm is the introduction of a modal operator to mark the special class of monotonic implications, those that *can* be arbitrarily weakened with further collateral premises. (That turns out to include all those that hold in virtue of the meanings of the logical connectives alone). The benefits of treating monotonicity as a modality are many, and the costs are few. Treating logic as built on and explicating (elaborated from and explicative of) material relations of consequence and incompatibility offers another option besides substructural logics, when relaxing global structural constraints. One can introduce logical vocabulary to codify fine-grained *local* structures. These *monotonicity-modal* expressivist logics implement technically a central methodological principle of expressivist logics: don't presuppose Procrustean *global* structural requirements on the material relations of consequence and incompatibility one seeks to codify logically. Instead, relax those global structures and introduce vocabulary that will let one *say explicitly*, in the logically extended object language, *that* they hold *locally*, wherever in fact they still do.

[7] Of course not everyone—relevantists, for example—will agree that contexted reflexivity *is* minimal structure. So it should be admitted that this is a contentious description.

The Pragmatist Roots and Some Expressivist Extensions
of *The Dialogical Roots of Deduction*

Bob Brandom

Dutilh Novaes's new book is both original and important.[1] *The Dialogical Roots of Deduction* investigates the relations between deduction and dialogue. Its approach is comprehensive, progressing along four interlocking, mutually-supporting dimensions: historical, philosophical, psychological, and in connection with mathematical practice. By doing that it substantially illuminates a number of distinctive features of deductive logical relations that philosophers of logic have found problematic or puzzling. These include the necessary *truth-preservingness* of deductive consequence relations, the irrelevance of the issue of whether or not one believes the premises and conclusions of deductive consequence relations, the distinctive sort of *perspicuousness* afforded by the possibility of unpacking deductive arguments into step-by-step chains, each of whose individual links is immediately cogent, and the nature of the *normative significance* of logical relations. There are substantial contemporary literatures devoted to each of these topics. But they are typically treated in isolation from one another. Perhaps the most impressive feature of the book, marking it as a landmark achievement in the field, is the fact that Dutilh Novaes offers a *systematic*, *unified* account that traces *all* of these phenomena back to the *same* source and persuasively explains them all on the *same* basis: the relation between deductive logical relations and dialogic practices.

I think it is particularly worthwhile to get clear about the nature of the central, weight-bearing relation between deduction and dialogue that Dutilh Novaes uncovers and elaborates by means of the metaphor of "roots." I want to begin by making a suggestion about how we might characterize *one* fundamental philosophical idea that animates this metaphor, in the hopes of clarifying its philosophical significance by connecting it to some other ideas. The idea I follow up on is that Dutilh Novaes shows us (among much else), how to understand relations of deductive consequence (what is expressed by the turnstile) in terms of dialogic practices. Then I want to consider one way of following out the clues suggested by that formulation, so as to generalize Dutilh Novaes ideas by applying them to areas beyond those in which she introduces them: from thinking about our peculiar and rarified deductive practices to thinking about reasoning in general.

[1] Catarina Dutilh Novaes, *The Dialogical Roots of Deduction: Historical, Cognitive, and Philosophical Perspectives on Reasoning* [Cambridge University Press, 2021].

I. Reason Relations and Reasoning Practices

Lewis Carroll's fable "Achilles and the Tortoise" vividly teaches us to distinguish, in John Stuart Mill's terms, "premises from which to reason" (including those codifying implication relations) from "rules in accordance with which to reason." He shows, in particular, that action-governing norms in the form of rules cannot be eliminated wholesale in favor of premises in the form of conditionals, on the pain of rational impotence. Gil Harman has radicalized this point, arguing for the initially astonishing conclusion that there are no such things as deductive rules of inference. If there were, presumably a paradigmatic one would would correspond to *modus ponens*, and would say something like: If you believe that *p* and you believe that *if p then q*, then you should believe that *q*. But, he points out, that would be a terrible general norm governing what one should *do*, inferentially, given those beliefs. For your collateral beliefs might give you much better reasons *against q* than you have *for* either *p* or *if p then q*. In that case, it seems, what one would have reason to *do* is to give up one of those commitments.

This argument points to the conclusion that we should distinguish between deductive logical *relations* of implication and incompatibility and inferential *activities* or *practices*. What logical relations establish deductively is that *p*, *if p then q*, and *not-q* are incompatible, because *p* and *if p then q* stand in the relation of *logical implication* to *q*, and *q* and *not-q* stand in the relation of *logical incompatibility* to each other. These deductive logical relations normatively *constrain* our reasoning activities. For they tell us that if we find ourselves committed to all of *p*, *if p then q*, and *not-q*, that we are in a normatively bad position. Those commitments are incompatible, we cannot be entitled to all of them. This entails a normative obligation to *do something*, to alter this normatively unsatisfactory situation. That *p* and *if p then q* imply *q* tells us that *in some sense, p* and *if p then q* together provide reasons *for q*. And that *not-q* is incompatible with *q* tells us that *in some sense not-q* provides a reason *against q*. But those reason *relations* do not *determine*, but only *constrain* what we are obliged to *do*, the *reasoning practices* or inferential activities of *changing* our commitments (and so entitlements) that we should engage in.

Deductive reason relations tell us about consequential relations among commitments, and about which commitments we can jointly be entitled to. The sense in which implication relations express what commitments provide reasons *for* what other commitments is something like that one is committed to the consequences of one's other commitments. And the sense in which incompatibility relations express what commitments provide reasons *against* what other commitments is something like that one is not entitled to commitments incompatible with one's other commitments. But the implicit norms that one should acknowledge the consequences of one's commitments and not undertake commitments to which one is not entitled can collide. When those norms do collide, the reason relations by themselves don't dictate what interlocutors should *do*. They do not provide definitive guidance for reason*ing*: making inferences that alter one's commitments and entitlements.

Dutilh Novaes discusses the question of how to understand the normative significance of deductive logical relations (and the large literature that has grown up around this topic downstream from Harman's initial delineation of it, and in a much more sharply focused fashion in the wake of MacFarlane's pathbreaking formulations of it) as one among a battery of issues in contemporary philosophy of logic she addresses. I would like to foreground this topic, and consider how her project looks if we use it as a lens through which to view the whole thing. Looking at it from this perspective highlights in particular two of the master-ideas articulated and developed in the book.

1. We should understand the nature and significance of the deductive reason *relations* traditionally studied under the heading of "deductive logic" in terms of the role those relations of logical consequence and incompatibility play in *practices* of reason*ing*.
2. Those reasoning practices should be understood as essentially *dialogical*: as practices of giving and assessing reasons, by defending and challenging commitments.

Building on her work over the past decades, Dutilh Novaes argues that the way of thinking about logic encapsulated in these two claims not only has a long history, but has substantial claims to be the *traditional* way of understanding the nature of logic. This traditional insight was obscured in the twentieth century by formalist models of uninterpreted calculi thought of as illuminating mathematical proofs, understood either as themselves formal objects, or at best as monological proof-procedures. It is part of Dutilh Novaes's argument that the latter picture was never true to actual mathematical practice.

Let me say a bit more about each of these big ideas, putting them in my terms, rather than hers. The first claim is an overarching methodological commitment that orients the entire project. It is what the image of "roots" in the title is metaphorical for. I understand it in terms of a broadly *expressivist* view about logic: the point of logic is to make explicit essential structural features of reasoning. Further, I think of it as pursuing a broadly *pragmatist* approach to logic. By this I mean that the order of explanation appeals first to *pragmatics*, thought of as the theory of the *use* of logical expressions (the practices in which it *does* matter what interlocutors are committed or entitled to, and how those statuses change), in order then to understand the deductive logical relations expressed by applying those expressions (for which questions of belief or entitlement are "bracketed").

I think it is interesting to consider this pragmatist order of explanation in connection with an *inferentialist* understanding of the meanings or conceptual contents of logical locutions. Inferentialists about logic think of that the meaning of logical connectives consist in the role they play in deductive logical relations. In natural deduction formulations, the difference between the meanings of conditionals, negation, and disjunction in classical logic and intuitionistic logic, for instance, are taken by inferentialists to consist in the difference between the pairs of introduction and elimination rules that introduce and define those connectives. Inferentialism about the meaning of logical vocabulary (the content of logical concepts) is a much more widely shared and, arguably, much more plausible thesis than general semantic inferentialism that aims to extend the inferential-role account of the meanings of logical vocabulary to empirical descriptive vocabulary and beyond. (A good account both of the logical inferentialist species and the semantic inferentialist genus, see Jaroslav Peregrin's masterful recent book *Inferentialism*.[2] One of the points he makes there is that although the origins of enthusiasm for logical inferentialism are to be found in those who prefer to think about logic in the proof-theoretic terms due originally to Gentzen rather than the model-theoretic terms due originally to Tarski, inferentialists can think of proof-theory and model-theory just as rival metalanguages in which to specify inferential roles.)

A weak, minimal sort of logical inferentialism claims that the meanings of logical locutions are articulated by the deductive logical relations compound sentences containing them stand in to one another. In the context of such a view, Dutilh Novaes first master-idea shows up as a form of *pragmatism* about logic. By 'pragmatism' here I mean an order of explanation that runs from *pragmatics*, the study of the *use* of linguistic expressions, to *semantics*, the study of the *meaning* of that vocabulary, the conceptual contents its use expresses. Logical inferentialism supplies the middle term connecting pragmatics with semantics in this sort of pragmatism about logic, by understanding the meaning or conceptual content expressed by logical vocabulary in terms of the role that vocabulary plays in the relations of logical consequence and incompatibility, which are in turn to be explained by appeal to the use of the vocabulary in practices of reasoning.

Dutilh Novaes's second master-idea is worked out in the form of a regimented dialogical pragmatics of reasoning that she calls the "Prover-Skeptic" model of dialogue. This model is structured by two complementary functional social roles. The job of the proponent is rationally to defend a claim, by giving reasons for it, and the job of the skeptic is rationally to prove and

[2] [Palgrave Macmillan, 2014]

challenge it, by trying out reasons against it. One of Dutilh Novaes's central claims is the striking observation that just because these in some sense *antagonistic* roles are *complementary* in the particular way they are, they can also be seen as aspects of a fundamentally cooperative endeavor: examining the credentials of the arguments they contest.

Dutilh Novaes introduces her deeply historically grounded pragmatic model by arguing for its superiority over the earlier regimentations of dialogues by Lorenzen and Hintikka. And her principal concern is to show how a battery of properties of logical consequence relations that the philosophy of logic tradition has found to be both central and perhaps essential to its understanding of logical consequence relations and also deeply puzzling are illuminated by looking at the role they play in dialogic explorations of the credentials of claims that accord with the regimented Prover-Skeptic model. Her attention is accordingly focused on the historical antecedents of the model (looking backward) and its explanatory benefits (looking forward). I want to raise and begin to explore a different sort of issue: how to understand why things *must* be as she shows them to be. This is to look in a certain sense at the roots of the roots, or, maybe better, the ground that supports the roots she had brought into view for us.

The Prover-Skeptic pragmatics is motivated by the idea that the reasoning practices on the basis of which we are to understand deductive logical relations (and so, I want to say—though she does not—the semantic contents of logical concepts) have a distinctive *dialogical* structure. They are practices of making claims and assessing the *reasons* for those claims. Assessing the *rational* credentials of claims essentially involves doing two different kinds of thing: defending claims by giving *reasons for* them and challenging claims by giving *reasons against* them. Giving a reason *for* a claim is asserting something that *implies* it or has it as a consequence. Giving a reason *against* a claim is asserting something that is *incompatible* with it or rules it out. Accordingly, the two fundamental reason relations of implication and incompatibility can be understood in terms of the roles they play in these two fundamental kinds of acts of reason-giving.

In Dutilh Novaes's Prover-Skeptic dialogic model of reasoning practices, those roles are complementary and interdependent. That does seem to reflect, in a suitably idealized and regimented way, features of our actual practices. And it does exhibit pragmatic roots, in our practices of giving and challenging reasons, for the basic relations that our logics codify: logical consequence and logical consistency. It thereby provides a satisfactory and illuminating answer to the question Harman raised, about how we should understand the relations between deductive logical *relations* and inferential *practices*.

Further, I think we can see some considerations that might be appealed to in what amounts to a transcendental argument for the necessity of two generic roles of which Dutlh Novaes's Prover and Skeptic are more specific versions. As I have characterized them above, these are the roles of *defending* a commitment by giving reasons *for* it, and *challenging* a commitment by giving reasons *against* it.[3] In order to think further about these roles—to think, as it were, about the (meta)role played in reasoning practices by these two roles interlocutors can play—we might consider alternatives. We could call reasoning practices that consisted exclusively of offering reasons *for* "dogmatic" reasoning practices, reasoning practices that consisted exclusively of offering reasons *against* "skeptical" reasoning practices, and reasoning practices that allow both "critical" reasoning practices. The Prover-Skeptic model regiments critical reasoning practices. What would be wrong with its impoverished merely dogmatic and merely skeptical cousins?

We can think of *inquiry* as what practically pursues the issue of whether to accept or reject a commitment. Inquiry is *rational* insofar as it consists of assessing *reasons* for adopting one of these practical attitudes of acceptance or rejection. Citing reasons is what *entitles* one to accept or reject a commitment. (In the whole picture, within a properly critical practice, the commitments will be properly understood as *propositionally contentful*, as *claimables*, insofar as they stand in reason relations of implication and (in)consistency with other claimables, in virtue of the role they play in critical practices of assessing their rational credentials.) Dogmatic reasoning practices, then, admit only the offering of reasons *for* commitments, that is, reasons to *accept* claims. In a wonderful essay called "Why 'Not'?", Huw Price considers the practical deficiencies of what I am calling "dogmatic" reason-giving practices.[4] He imagines "ideological positivists," who do not have a way of denying or rejecting a claim. They lack any practical acknowledgment of the *incompatibility* of two claims. (It will follow that in their logic they have no way of *negating* a claim—hence the issue of his title.) He illustrates why such practices wouldn't work with a nice dialogue:

Me: 'Fred is in the kitchen.' (Sets off for kitchen.)
You: 'Wait! Fred is in the garden.'
Me: 'I see. But he is in the kitchen, so I'll go there.' (Sets off.)
You: 'You lack understanding. The kitchen is Fred-free.'

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[3] For the sake of my argument, I am abstracting away from the details of Dutilh Novaes's model in a way that might be thought to falsify the characterization. In particular, I am ignoring the Skeptic's role as granting or failing to grant premises, and as objecting to moves specifically by offering counterexamples.

[4] Huw Price "Why 'Not'?" Mind, New Series, Vol. 99, No. 394 (Apr., 1990), pp. 221-238. Published by: Oxford University Press on behalf of the Mind Association. Stable URL: https://www.jstor.org/stable/2254961. Dialogue quoted is from p. 224.

Me: 'Is it really? But Fred's in it, and that's the important thing.'
(Leaves for kitchen.)

Unless our commitments can *exclude* some other commitments, *preclude* our *entitlement* to those other commitments, they can't guide our actions. We would learn *nothing* practically from finding out that someone is entitled to a commitment—say, "Fred is in the garden,"—unless it meant that one could *not* be entitled to some other commitments—"Fred is in the kitchen,"—which accordingly count as *incompatible* with it. The very same reason that is a reason *for* one commitment must also be a reason *against* some others. The very same reason entitling one to *accept* a commitment must be a reason obliging one to *reject* those others, in the sense of *precluding entitlement* to them.

This parity of reasons for and reasons against, the requirement that reasons to accept one commitment *must be* reasons to reject some others, is acknowledged in the Scholastic slogan adopted and exploited by Spinoza, "*omnis determinatio est negatio.*" And it is at the heart of modern information theory, in the form of its founder Claude Shannon's idea that the information conveyed by a signal can be measured by the extent to which it *rules out* practical options on the part of the recipient compared to the situation before receipt of the signal. In monologic conceptions of mathematical proof, this fundamental oppositional dimension of reasoning is represented by the felt obligation to show the *consistency* of the conclusions one has derived, the theorems one has proven from the axioms, given the rules. Showing consistency is precisely showing that none of the conclusions one has arrived at *contradict* any of the others, in the sense that commitment to one precludes entitlement to the other. In this way the role of the Skeptic in Prover-Skeptic dialogues is internalized into the monological procedure. But this essential feature of the reasoning is also *hidden* in the monological conception of deductive practice, left *implicit* in the requirement of consistency of theories. It is made explicit by the dialogic division of rational labor in the Prover-Skeptic model.

Appreciating the transcendental practical necessity of incorporating reasons against (reasons to reject) in dogmatic reasoning practices also points us to the considerations that let us dispose pretty quickly of the notional possibility of purely skeptical reasoning practices: those that would admit only reasons against, reasons to reject commitments. Dogmatic practices practically erase the distinction between commitments interlocutors are entitled to and those they are not entitled to by treating commitments as promiscuously compatible. Skeptical practices practically erase the distinction between commitments interlocutors are entitled to and those they are not entitled to by offering no way at all for interlocutors to become entitled to their commitments. We could imagine that these skeptical reasoning practices take place against a background where interlocutors count as entitled to whatever commitments they make by default. Then the practices are Popperian: the practice of reasoning is the process of falsifying commitments, unmasking them by showing we are *not* entitled to them. Presumably, the reasons against are *also* ones that are "innocent until proven guilty," in that those who offer such reasons count as entitled to them until and unless *those* entitlements are undercut by contrary assertions. Here, too, though, it is hard to see that it makes any difference at all which commitments interlocutors are entitled to. There is no way to transmit such entitlements to further commitments. The *only* use one can make of default entitlements that have not yet been infirmed is to infirm other commitments. Perhaps the practice is that discredited commitments lose their capacity to discredit others. But so what? *Any* incompatible claim can be made to attack a given commitment, and so long as it has not yet been discredited by contrary assertion, it will count as disconfirmatory.

Reasoning is essentially about the rational *credentials* of possible commitments, which is to say about *entitlements* to those commitments. It cannot consist entirely of obligatory movements from commitments to consequential commitments. Reasons must include reasons entitling one to accept or to reject further commitments. Reasoning exploits relations linking entitlement to one commitment to entitlement to others. (That is why the "roots" of relations of logical consequence and (in)consistency are to be found in reasoning practices.) Entitlement to a commitment must be consequential for entitlements to other commitments; it must be transmissible. And it must entitle one *both* to *accept* some further commitment *and* to *reject* some other commitments. Dogmatic or skeptical reasoning practices either grow entitlements wildly and without limit, or shrink them wildly and without limit. Only critical reasoning practices can flourish. To use a Hegelian image, both the inhalation and the exhalation of reasoning are essential to its life and health. The stylized roles of Prover and Skeptic, Proponent and Opponent, in Dutilh Novaes's model of reasoning are accordingly idealizations distilling features essential to any practices recognizable as *reasoning*. Ultimately, they reflect the essential dual role played by the choice of accepting or rejecting commitments, including practical commitments to *do* something. Reasons transmit entitlements to those attitudes, and so essentially, and not just accidentally come in the two flavors of reasons for and reasons against commitments. One central contribution of Dutilh Novaes's book is its working out of the pragmatist idea that logical reason *relations* of implication and inconsistency are to be understood in terms of the role they play in *practices* with this essentially dual structure, which is internal to the idea of reasoning as such.

II. Material Reason Relations and Substructurality

Notice that everything I said in the previous discussion of the transcendental pragmatic reasons for the Prover-Skeptic model to have the structure it does were addressed to practices of giving and challenging *reasons* generally. There was no restriction to practices of offering *logically* good reasons to accept or reject claims. I want not only to offer further grounding for Dutilh Novaes's ideas about the dialogic pragmatic roots of deductive logical reason relations (which she might well think is not needed), but also to suggest how those ideas might be developed further (albeit in directions she might well not be willing to take them).

I want to argue that at least one of Dutilh Novaes's lines of thought can be extended from specifically *logical* reason-relations and their rootedness in dialogic practice to reason-relations *generally*. And, as a consequence, I want to extend that idea from reason-relations that meet the traditional Tarski-Gentzen structural conditions of monotonicity and transitivity to reason-relations that are radically *sub*structural, in the sense that the do *not* satisfy those structural conditions. And I want to claim that it is a signal indication of the depth and power of the "rootedness in dialogue" idea that it *does* continue to apply in this radically extended domain.

Relations of logical consequence depend essentially on the occurrence of specifically *logical* vocabulary in the premises and conclusions. (Logical inferentialists believe that that is because such relations articulate the conceptual content expressed by logical vocabulary. But I am claiming only something weaker here.) We can see that, because logically good implications remain good upon arbitrary substitution of nonlogical for nonlogical vocabulary (and logical contradictions also remain logical contradictions under such substitutions). This has led many to conclude that deductive reason relations hold in virtue of the logical form of the sentences involved. It is one of innovations and signal advantages of Dutilh Novaes's account that she does *not* take its *formality* to be a defining characteristic of deductive logical relations of consequence and inconsistency. Her view is that

> Rather than being that in virtue of which an argument is deductively valid, logical forms/schemata are in fact convenient devices that allow us to track deductive validity with less effort (though for a limited range of arguments). [*DRD* 18]

In this way, many false trails and needless complications are avoided.[5]

[5] John MacFarlane's 2000 Pittsburgh Ph.D. dissertation "What Does It Mean to Say that Logic is Formal?" is a conceptually wide-ranging and historically informed discussion of the many ramifications of the issue of the formality of logic that has been vastly and justly influential (in spite of never having been officially published).

One need not take on that traditional commitment to deductive reason relations holding in virtue of their logical form, however, in order to accept the weaker claim that those relations depend essentially on the occurrence of logical vocabulary in the claimables they relate, in the sense of their being substitutionally robust in the way just described. And that observation opens the way to the realization that we can start with the observation that there least appear to be implications and inconsistencies that essentially depend in a corresponding way on the occurrence of *non*logical vocabulary.

Pittsburgh is to the south of Montreal.
therefore
Montreal is to the north of Pittsburgh.

is an implication that depends essentially on the contents of the concepts expressed by the nonlogical terms "south" and "north," and is robust under arbitrary substitution for the terms "Pittsburgh" and "Montreal." And the fact that the set of claims

{Monochromatic surface A is green, Monochromatic surface A is red}

is inconsistent essentially depends on the contents of the concepts expressed by the nonlogical terms "green" and "red" and "monochromatic," and is robust under substitutions for the place-holder "A." Wilfrid Sellars calls these "material" implications and inconsistencies. The term should be thought of as contrasting with "logical." (Sellars probably picked it because he *did* think of logic as essentially formal. But acknowledging the contrast his terminology marks does not depend on that collateral commitment.)

If we now ask how reason relations of consequence or implication and inconsistency or incompatibility that essentially depend on the occurrence of specifically *logical* vocabulary (in the sense defined by the method of noting invariance under substitution) should be thought of as standing to reason relations of implication and incompatibility that essentially depend on the occurrence of *non*logical vocabulary, another traditional philosophical thesis about logic comes into view. This is *logicism* about reason relations generally. It claims that *all* good reasons are ultimately to be understood as *logically* good reasons. What it is for some claims *rationally* to imply another, or to be *rationally* incompatible with it, is for them *logically* to imply or be *logically* inconsistent with it. The substantial commitment that is fundamental to this sort of approach is what Sellars calls

> ...the received dogma...that the inference which finds its expression in "It is raining, therefore the streets will be wet" is an enthymeme.[6]

6 Sellars "Inference and Meaning," reprinted in *Pure Pragmatics and Possible Worlds* J. Sicha (ed.) [Ridgeview Publishing Co. 1980] (hereafter, *PPPW*), pp. 261/313.

According to this line of thought, wherever an inference is endorsed, it is because of belief in a conditional. Thus the instanced inference is understood as implicitly involving the conditional "If it is raining, then the streets will be wet". With that "suppressed" premise supplied, the inference is an instance of the formally valid scheme of conditional detachment.

Logicism about reasons generally is a very strong claim. For it is committed to the open-ended program of reconstructing all good reasons as logically good. It faces well-known difficulties even on its most friendly ground of mathematical reasoning. But the logicist program faces even larger challenges in addressing the reason relations that show up for instance in practical reasoning (such as jurisprudential reasoning), inductive scientific or probabilistic Bayesian reasoning, and the sorts of implication and inconsistency appealed to in informal conversation. Further, logicism mut understand any and every grasp of relations of being a reason for or reason against as an essentially *logical* ability. The grasp of logic attributed must be implicit, since it need not manifest itself in any capacity to complete or fill in “enthymematic” reasoning, manipulate specifically logical vocabulary, assess logical derivations, or distinguish logical tautologies. It is hard to see how such an occult logical ability can be specified in terms sufficiently non-question-begging to do any actual explanatory work.

Dutilh Novaes is not committed to logicism about reasons generally. Her topic is restricted to deductive logical reasons. And acknowledging that relations of rational implication and incompatibility can depend essentially on the presence of logical vocabulary *or* the presence of nonlogical vocabulary in no way requires making the additional reductionist logicist claim about the relations between these two cases. We can simply observe that in addition to logical consequence and inconsistency there are *material* relations of implication and incompatibility. Implications and incompatibilities can depend essentially on the occurrence of particular bits of vocabulary in them, as assessed by substitutional tests. Some of that vocabulary is logical, and its occurrence is essential to the logical goodness of reasons for and against, and some is not, and its occurrence is essential to the material goodness of reasons for and against.

Note that a question that remains even after we have adopted this relaxed, nonlogicist attitude toward the relations between logically good reasons and materially good reasons (understood in terms of their ranges of substitutional robustness) concerns the distinction between logical and nonlogical vocabulary, on which it depends. This is the *demarcation* issue in the philosophy of logic: how to distinguish specifically logical vocabulary, or the concepts it expresses. We logical expressivists, who are also semantic inferentialists, take it that logical vocabulary is distinguished by playing a distinctive expressive role. That role is to make explicit the reason relations of consequence and incompatibility that articulate the contents of *all* concepts—beginning with the *material* relations of consequence and incompatibility that articulate the contents expressed by *non*logical vocabulary. The expressive job characteristic of conditionals is to make implication relations explicit in the (logically extended) object language, and the expressive job characteristic of negation is to make incompatibility relations explicit in the object language. Dutilh Novaes does not explicitly address the demarcation question. In practice, she adopts a historical-developmental approach, rather than a strictly functional one.

However it is with logical consequence, *material* consequence relations do not in general satisfy the strong structural conditions that have traditionally been thought to be essential features of specifically *logical* consequence. In particular, as Dutilh Novaes acknowledges, ordinary reasoning, by contrast to deductive reasoning, is often nonmonotonic. Adding further premises can turn good implications into bad ones. As a result, ordinary reasoning admits the construction of Sobel sequences, where the addition of further considerations flips the polarity of implications in both directions:

- If I strike this dry, well-made match, then it will light.
- If I strike this dry, well-made match in a strong magnetic field, then it will not light.
- If I strike this dry, well-made match in a strong magnetic field but inside a Faraday cage, then it will light.
- If I strike this dry, well-made match in a strong magnetic field but inside a Faraday cage, and in a room from which the air has been evacuated, then it will not light.
- …

And so on.

One *might* insist that all the implications codified in these conditionals are strictly false, expressing enthymematic approximations of the true conditionals that would explicitly include as premises *all* the potentially defeating or enabling conditions. (I would caution that *if* one takes such a line, one should *not* do so because of an implicit commitment to logicism about the goodness of material implications.) But speaking against such a conception is the suspicion that there is no definite totality of such defeating and enabling conditions, or that if there were, it would in any case not be finitely statable.

Even rigorous reasoning in mature sciences depends for its cogency on tacit assumptions it would be at least tedious and possibly simply impossible to state explicitly. When I apply a bit of ideal theory, say Ohm's law relating current, voltage, and resistance, in order to make predictions about what various meter-readings will be if I make an intervention in an electrical circuit, any inferences I make will be defeasible by a host of potential confounding collateral circumstances, such as all the sorts of defects there could be in the measuring apparatus. Medical diagnosis consists largely of making inferences from history and physical findings that are then found to be defeated by conditions revealed by further tests. (This is the plot of every episode of medical shows such as "House.") And legal reasoning in trials, both civil and criminal, depends essentially on the making of rebuttable presumptions and the drawing of rebuttable conclusions.

The features of these practices that acknowledge in-principle defeasibility by further auxiliary hypotheses serving as collateral premises are not avoidable conveniences of reason-giving practices. They stem from the necessity for finitely statable arguments in the face of infinite possibilities for collateral information that would infirm the implications in question. Our empirical reasoning cannot avoid what is explicitly acknowledged by the use of *ceteris paribus* clauses. It is not that appending such a clause to an implication magically turns a defeasible implication into an indefeasible one. (Latin phrases whose utterance can make that sort of difference are called "spells.") The expressive function of *ceteris paribus* clauses is just to acknowledge explicitly the defeasibility of an implication. (On pain of triviality, it can't be that its force is "*q* follows from *p*, except in cases where it doesn't.")

III. Substructural Material Reason Relations Support Well-Behaved Logics

Pointing out the substructural character of material implication (and incompatibility) relations risks severing the connection with logic entirely. What reason is there to think of them as relations of *rational consequence* at all. After all, Tarski had good reason to think his structural conditions were minimal conditions on consequence *tout court* We can imagine Tarski or Gentzen saying: When we say that these are *reason* relations, relations of implication, consequence, or rational following from, the best evidence for *our* claim is that you can do *logic* with them. Even if you don't think that doing is *all* there is to reasoning (for instance, because you think being able to engage in dialogic reasoning practices is important, too), still, we can argue that it is a *necessary* condition of being *reason* relations that they articulate a *logic*.

And highlighting their substructurality, the failures of monotonicity (and, although I have not gone into it here, even transitivity) makes it looks as though *material* consequence relations have nothing to do with logic at all. It is one thing to object to reason relations such as material consequence relations being *reducible* to logical reason relations; that is to reject logicism. It would be a *much* stronger claim that a relation is intelligible as being a *rational* consequence relation if it has *nothing* to do with, no principled relation to, logic and *logical* consequence relations.

The way I would like to put the challenge ("as the one playing the role of Skeptic says to the one playing the role of Prover") is this: The claim that substructural material relations of, as it were, implication and incompatibility really qualify as *reason* relations, in the sense of underwriting relations of being a reason *for* and being a reason *against*, depends on, has as a necessary condition, standing in the right relation to *logic*. To assess the claim in the light of that challenge, we need to settle what the "right relation to logic" is. I have already rejected the logicist reading of it, which fixes the <u>logic</u> side in advance, and then treats as rational, as *reason* relations, only what can be reconstructed in terms of specifically *logical* relations of deductive consequence and inconsistency. And I suspect that there is no way of answering the question of what the relation is that holds between logic and the reason relations that codify what is a reason for or against what that is neutral across widely varying philosophies of logic. So I will address it from the point of view of what I take to be the correct answer to the question.

Logical expressivists are in a way the mirror-image of logicists about how to understand the relations between logic and the relations of being a reason for and being a reason against. Where logicists think logic *determines* what premises provide reasons for a claim and what premises provide reasons against it, we logical expressivists think the expressive role distinctive of logical vocabulary is to let us *say* what premises provide reasons for a claim and what premises provide reasons against it. We *start* with reason relations, and introduce logical vocabulary to *express* them. Conditionals let us express implication relations in the form of claimables that can both serve as and stand in need of reasons and so be rationally supported and challenged. And negation does the same thing for relations of incompatibility.

So for us expressivists, the question of whether substructural material relations of consequence and incompatibility qualify as genuine *reason* relations in virtue of their relation to logic (admittedly not the only consideration that bears on the larger question of being reason relations) comes down to the question of whether they are codifiable in logical terms in a way that is both formally tractable and recognizably similar to traditional logics. To this question we can respond with a resounding "Yes." Recent work by Ulf Hlobil and Dan Kaplan, in our research group "Research on Logical Expressivism" (ROLE) has shown how to build well-behaved logics on top of substructural relations of material implication and incompatibility.[7]

The idea is to begin with what we call a "material semantic frame" (MSF) defined on a language L_0 consisting of a finite set of logical atoms. Such a frame consists of a consequence relation $|\!\sim_0$, and a distinguished set of sets of atomic sentences that are treated as incoherent. There are both single-succedent and multi-succedent sequent calculus versions of the logic, but I'll start by talking just about the single-succedent case. Then we can encode the material incoherence of a set $\Gamma \subseteq L_0$ as a sequent: $\Gamma |\!\sim_0 \perp$.

We impose only two minimal structural conditions on the base MSF: contexted reflexivity or containment (CO) and a principle we call "*ex falso fixo quodlibet*" (ExFF). The first says that for any set of sentences Γ and any sentence A in L_0,

CO: $\Gamma, A |\!\sim_0 A$.

A is a material consequence of any set of premises that contains A. The second is a version of explosion or *ex falso quodlibet* adapted for a nonmonotonic setting. It can happen that although Γ is incoherent, it is defeasibly so, in that adding some further sentences to it yields a coherent set. We mark only the *indefeasibly* or persistently incoherent sets by requiring that

ExFF: $\forall(A \in L_0)\ [\forall(\Delta \subseteq L_0)\ \Gamma,\Delta |\!\sim_0 \perp\ \Rightarrow \Gamma,\Delta |\!\sim_0 A]$.

Persistently incoherent premise-sets materially imply everything.

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7 Ulf Hlobil, "A Nonmonotonic Sequent Calculus for Inferentialist Expressivists," [In P. Arazim & M. Danˇcák (Eds.), The Logica Yearbook 2015 (pp. 87–105). College Publications.] and Daniel Kaplan, "A Multi-Succedent Sequent Calculus for Logical Expressivists," [In P. Arazim & M. Danˇcák (Eds.), The Logica Yearbook 2017. College Publications.].

We do *not* require that material semantic frames have consequence relations or incoherence properties that are monotonic. That is, we do *not* require:

MO: $\forall(A\in L_0)\forall(\Gamma,\Delta\subseteq L_0)$ [$\Gamma \mid\!\sim_0 A \Rightarrow \Gamma,\Delta \mid\!\sim_0 A$].

And we do *not* require that the consequence relations be transitive. That is, we do *not* require that Cut holds:

CT: $\forall(A,B\in L_0)\forall(\Gamma,\Delta\subseteq L_0)$ [($\Gamma \mid\!\sim_0 A$ & $\Gamma,A \mid\!\sim_0 B) \Rightarrow \Gamma \mid\!\sim_0 B$].

We can extend the atomic base language L_0 in the usual way, by adding sentential logical connectives to produce a language consisting of logically complex sentences formed by applying those connectives recursively to the language L_0 of logical atoms. Hlobil and Kaplan show how to use Gentzen-style sequent calculus connective rules to introduce specify the consequence and incompatibility reason relations (relations of being a reason for and being a reason against) of (sets of) sentences in the logically extended language.

As expressivists about the functional roles that demarcate specifically *logical* vocabulary, we want to impose two crucial restrictions on the connective rules defining conditionals and negations. Since we want conditionals to codify implication relations (including material ones), we want the conditional operator to satisfy the Ramsey condition, in both directions:

Ramsey Condition: $\boldsymbol{\Gamma\mid\!\sim A\rightarrow B}$ **iff** $\boldsymbol{\Gamma,A\mid\!\sim B}$.

That is, a premise-set implies a conditional just in case the result of adding the antecedent to that premise-set implies the consequent. A conditional that satisfies this equivalence can be called a "Ramsey-test conditional," since Frank Ramsey first proposed thinking of conditionals this way.

Since we want negation to codify incompatibility relations (including material ones), we want the negation operator to satisfy the Minimal Negation condition, in both directions:

Minimal Negation Condition: $\boldsymbol{\Gamma\mid\!\sim\neg A}$ **iff** $\boldsymbol{\Gamma,A\mid\!\sim\bot}$.

That is, a premise-set implies not-A just in case A is incompatible with that premise-set. (It follows that $\neg A$ is the minimal incompatible of A, in the sense of being implied by everything that is incompatible with A.)

Underwriting these *bi*conditionals requires connective definitions that are reversible. Gentzen's student Ketonen showed how to reformulate Gentzen's connective definitions so as to make them reversible. We adopt his formulations. It turns out that in order to extend the underlying MSF governing logically atomic sentences conservatively, we need to mix and match additive and multiplicative rules for conjunction and disjunction. (Otherwise, monotonicity gets built in.) In spite of those distinctions, our connective definitions, like Ketonen's originals, are fully equivalent to Gentzen's, in the sense that there is a derivation of a conclusion from some set of axioms using Ketonen's connective definitions just in case there is derivation of that conclusion from the same set of axioms—in their system, always instances of Reflexivity: A|~A (we use Contexted Reflexivity or Containment, CO)—using Gentzen's connective definitions.[8]

In this way, new consequence and incompatibility relations are defined for the logically extended language by deriving them from more basic, nonlogical relations of consequence and incompatibility. Because the extension is conservative over the underlying material semantic frame—in that the implications and incompatibilities involving only logically atomic sentences are the same in the extension as in the original—the new reason relations will not in general be monotonic or transitive. Nonetheless, the result yields traditional intuitionistic and classical logics as limiting cases.

This is so in three different ways.

- First, the resulting system is supraclassical (in the multisuccedent case) or supraintuitionistic (in the single-succedent case)—that is, they validate all the classical or intuitionistic implications, respectively.
- Second, they yield straightforwardly classical and intuitionistic consequence relations (in the multisuccedent and single-succedent cases, respectively) if the MSFs they extend are "flat"—that is, consist exclusively of instances of Contexted Reflexivity or Containment, of the form Γ, A|~A.
- Third, the purely *logical* portion of the implication and incompatibility relations defined over the logically extended language are fully structural, and indeed, are just the classical and intuitionist relations of consequence and inconsistency. By "purely logical" I mean the consequences that hold upon arbitrary substitution of nonlogical for nonlogical vocabulary. In our systems, these are the same consequences and incompatibilities that hold no matter what underlying material semantic frame one extends logically.

The substructural character of prelogical relations of consequence and incompatibility accordingly presents no bar to codifying them logically: using conditionals to express implications and negation to express incompatibility. And the logics that result are both well-behaved and familiar. So someone who, while rejecting the logicists' identification of *all* reason relations with *logical* reason relations—someone who thinks that *good* reasons for and against need not be exclusively *logically* good reasons for and against—nonetheless thinks both that

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8 The Ketonen rules we use are "mixed": rules with two top sequents are additive and rules with a single top sequent are multiplicative. This is required to avoid the connective rules forcing structural monotonicity in the extended language. It is very close to the system called 'G3cp' by Negri, Von Plato, and Ranta (2008), but with material axioms.

logic provides paradigmatic reason relations and that logic stands in some special and distinctive relation to reason relations generally has no reason to take the nonmonotonicity and nontransitivity of material relations of consequence and incompatibility to entail that they are not genuine *reason* relations. The fact that logical relations of being a reason for and being a reason against satisfy strong structural constraints does not speak against the substructural relations that articulate the contents of ordinary, nonlogical concepts being genuine reason relations. One can even do logic with them.

IV. Dialogues and Substructural Material Reason Relations

The argument I have just rehearsed addressed potential objections to acknowledging nonlogical, material relations of consequence and incompatibility as genuine reason relations, as being rational relations of supporting and ruling out conclusions, on the basis that they are unlike *logical* reason relations in being in general nonmonotonic and nontransitive. I did not consider the crucial new set of considerations that the *Dialogical Roots of Deduction* has put on the table. I addressed only a traditional requirement on being genuine reason relations**:** that one be able to do logic with them. Dutilh Novaes digs deeper. For she has an account of *why* you can do logic with genuine reason relations: because of the roles they play in dialogic practices of defending claims by giving reasons for them and challenging claims by giving reasons against them. What qualifies something as a reason relation is the functional role they play in dialogic practices of reason*ing*. Reason *relations* are to be understood in terms of reasoning *practices* having the dialogic form regimented in the Prover-Skeptic model.

I understand the book as saying (again, among *much* else) that if you want to understand what the turnstile expressing logical consequence *means*, what you are *saying* when you say that A is a deductive consequence of Γ, you have to look to the role that reason relation plays in dialogic practices of reasoning—practices that, according to the Prover-Skeptic model, are practices of defending claims by giving reasons *for* them and challenging claims by giving reasons *against* them.

According to this line of thought, the question we should be asking is whether the substructural character of material, nonlogical relations of consequences and incompatibility prevents them from playing a proper role in suitable analogues of Prover-Skeptic dialogues. If and insofar as they can play a functional role in such dialogues that is recognizably the same as that of fully structural, logical relations of deductive consequence (and inconsistency), they will qualify as genuine reason relations.

To determine the answer to this question, my colleague, Pitt doctoral student Yao Fan wrote a Python program to implement Prover-Skeptic dialogues based on substructural material semantic frames. We looked only at dialogues involving giving reasons for and against (defending and challenging) claims within the logically atomic, base language. After all, we know how what happens at that ground level completely determines what happens in the language that has been extended by the introduction of logical vocabulary—including the fully structural relations of *logical* consequence and inconsistency that result.

For demonstration purposes, we work in an artificial language with only 7 sentences: {a_1, a_2, a_3, a_4, a_5, a_6}, often abbreviated just by their subscripts. We define an arbitrary substructural *material semantic frame* (MSF) by specifying a material relation of *consequence* between premise-sets and single-sentence conclusions and a property of material *incoherence* that characterizes some sets of sentences. A conclusion is treated as *incompatible* with a premise-set iff their union is incoherent. MSFs accordingly codify *reason relations* of implication and incompatibility that will function dialogically to define relations of being a reason *for* and being a reason *against*.

Appendix 1 displays a sample MSF defined on that language of 7 logical atoms. It has 190 significant material implications, listed at lines 9ff.. Breaking these out, if you look at lines 70 to 75, listing the premise-sets that imply sentence 2, you will see that although 1 implies 2 and 3 implies 2, {1,3} does not imply 2, though {1,3,4,5,6} does. The material consequence relation is accordingly nonmonotonic. The MSF contains 60 materially incoherent sets (out of the 2^7=128 possible subsets of the language). These, too, are nonmonotonic. So looking at lines 147 to 154, listing the premise-sets that are incompatible with sentence 2, 3 is incompatible with 2 and 4 is incompatible with 2, but {3,4} is not.[9]

Appendix 2 displays one sample dialogue conducted on the basis of a sample material semantic frame like the one in Appendix 1. (For some of our experiments, we run tens of thousands of dialogues based on each MSF.) The leftmost column, after the line numbers, is the turn number of the dialogue. This one is 45 steps long. The next column lists the participants responsible for each move. The names we have given to our versions of Dutilh Novaes's "Prover-Skeptic" are "Claimant" (CL) and "Critic" (CR). They alternate. The next column to the right indicates the move that is challenged or defended by the move currently being made. Next is the *pragmatic significance* of the move made. I will return to that when we walk through the dialogue. The next column lists the move. It is always a reason *for* a claim, marked with "entails," or a reason *against* a claim, marked with "excludes." The next column, marked CL_AC, lists what the *claimant* (CL) is *committed* to *accept* after the move in that row. The column to its right lists what the *claimant* is *committed* to *reject* after that move. The next two

[9] At the very beginning of Appendix 1 you will see a long code that makes this MSF recoverable (for repeated dialogic experiments). It gives some indication how many MSF meeting our minimal structural constraints there are, even for the very basic 7-member material base language of our toy example. The set of incoherent sets is an element of the powerset of the powerset of the language, which has 2^128 elements. Even removing permutations, that is more than 10^30 possibilities. And there are even more ways to pick the material implications.

columns record what the claimant is *entitled* to *accept* and *entitled* to reject. The last four columns give the same information for the *critic* (CR).

This dialogue begins with the claimant (CL) putting forward a *proposal*. The dialogue consists of an inquiry into the defensibility of that proposal, given the underlying MSF. In our example, the claimant offers a reason for proposition a_2. That reason for is a set of premises that materially imply a_2, according to the MSF. All moves in the dialogue, whether giving reasons defending a claim or reasons challenging a claim, are drawn from the governing base MSF, which is understood as the common semantic basis of the dialogue. Making that move commits the claimant CL to all the premises, and to the conclusion. And, in the absence—thus far—of any challenge, under our rules the claimant counts as at this point entitled by default to those commitments. (The dialogues proceed according to what in *Making It Explicit* I call a "default-and-challenge" structure of entitlement.)

There are two ways to challenge a reason offered in any move in the dialogue. One can offer a reason *against* one of the premises, or a reason challenging the conclusion. If the conclusion is an endorsement (what is challenged was a reason for, an implication, marked by "entails") this will be a reason *against* ("excluding") the conclusion. If the conclusion was itself a rejection (an "exclusion"), then one challenges that conclusion by offering a reason *for* it. These are the "premise challenges" and "conclusion challenges" listed in the column of pragmatic significances ("PragSig") of the moves. In our example, in Turn 1 the critic CR challenges one of the premises of the proposal, by offering a reason *against* a_4. Making this move commits and default entitles CR to accept the premise and to reject the conclusion of the challenging reason. It does not alter the claimant's commitments, but does remove entitlement to the challenged premise and the conclusion it supports, while leaving intact the default entitlement to the other premises of the proposal.

The dialogue proceeds by claimant CL challenging the conclusion of CR's premise challenge, offering a reason against it. Since CR offered a reason *against* premise a_4 of the proposal, CL responds by defending that premise, offering a reason *for* it. This removes CR's entitlement to the conclusion of the challenge in Turn 1. It expands the claims CL is committed to accept, adds new default entitlements to those commitments, and restores CL's entitlement to the challenged premise. The interlocutors can challenge or defend any of the previous moves, not just the immediately preceding move. Notice that at Turn 5, the critic CR abandons the argument over the proposal premise a_4 and mounts a new challenge directly to the original proposal, by offering a conclusion challenge to it: a reason *against* its conclusion a_2.

The dialogue continues in this way until one interlocutor can no longer find in the MSF a reason it is eligible to put forward, defending or attacking the proposal, given its current commitments to accept and reject claims. In this case, at the close of the dialogue, the claimant CL has not managed to sustain entitlement to the conclusion of the proposal, a_2. The proposal is accordingly defeated. This is the scorekeeping outcome of the competitive aspect of the dialogue: either the proposal is vindicated and CL wins, or it is defeated, and CR wins. However, the *point* of the dialogue, toward which the activity of both interlocutors is directed, is investigating the credentials of the proposal. One crucial scorekeeping expression of the cooperative aspect of the enterprise is that the interlocutors have established a significant *common ground*. At the end of the day, they are *both* committed *and entitled* to propositions a_0, a_1, a_3, and a_5. One of our interests in this project lies in investigating connections between features of the underlying MSF and the emergence of common ground in dialogues investigating the credentials, the defensibility, of different proposals. (And a second phase of the project, inspired by Girard's ludics, goes the other way around, deriving the reason relations codified in MSFs from dialogues.) But all that is a topic for another time.

The principal conclusion I want to draw here from our experiments is that they *show* that substructural material semantic frames corresponding to material reason relations qualify as *reason* relations in the "dialogically rooted" sense: they play the right role in dialogic reasoning practices. The fact that they are nonmonotonic and nontransitive does not in any way disqualify them from supporting dialogues cooperatively-competitively investigating the credentials of claims by defending them with reasons and challenging them with reasons. I understand Dutilh Novaes as saying that if you want to understand what the turnstile expressing logical consequence *means*, what you are *saying* when you say that A is a deductive consequence of Γ, you have to look to the role that reason relation (plus inconsistency, I want to say) plays in dialogic practices of reasoning, practices that, according to the Prover-Skeptic model are practices of defending claims by giving reasons for them and challenging claims by giving reasons against them.

I propose that Dutilh Novaes's articulation of a tight connection between dialogic reasoning practices, having the Prover-Skeptic structure of giving and assessing reasons, on the one hand, with the paradigmatic reason relations (implication and incompatibility) of classical deductive logic, argues for the intelligibility of radically substructural *material* reason relations of implication and incompatibility, on the basis that such relations support dialogic reasoning practice that have the Prover-Skeptic structure of giving and assessing reasons. Further, I take it that she has explicitly left room for some such consequence of her views. She says, for instance:

> I submit that a plethora of kinds of dialogues should be embraced, and insofar as different logical systems will correspond to different kinds of dialogues, we end up with different, equally legitimate logical systems. What defines which logic is the 'right one' are the motivations of participants when engaging in a given dialogical situation, and their mutual agreement in terms of the structural and logical features of that particular conversation.

and

> Rather than entailing an overly permissive 'anything goes' attitude, the dialogical perspective in fact allows for the formulation of restrictions on what can count as a legitimate logical system: one that corresponds to a plausible kind of dialogue that people may actually feel compelled to engage in (though admittedly much work remains to be done on specific criteria of adequacy for dialogical systems).[10]

[10] Both passages from section 2.4 of Chapter Four of *DRD*.

V. Conclusion

The aim of this essay has been to place the achievement of the *Dialogic Roots of Deduction* in a larger philosophical context. That wider context encompasses a number of claims. At their core is the idea that what I called "reason *relations*" of implication and incompatibility mediate between essentially dialogic reasoning *practices* and the introduction of specifically logical vocabulary, and so relations of deductive *logical* consequence and inconsistency. Implication relations are to be understood dialogically in terms of the functional role they play in providing reasons *for* claims, by appeal to which those claims can be rationally defended. Incompatibility relations are to be understood dialogically in terms of the functional role they play in providing reasons *against* claims, by appeal to which those claims can be rationally challenged. And on the basis of reason relations so understood in terms of their role in reasoning, we can introduce specifically *logical* vocabulary that lets us make those relations explicit in a logically extended object language. The connective definitions that perform that distinctive expressive job then underwrite specifically *logical* relations of consequence and inconsistency relating logically complex sentences. And those logical reason relations underwrite properly deductive proofs.

The superstructure of *logical* reason relations exhibits the classical Tarski-Gentzen structural features, including monotonicity and transitivity, even if the underlying *material* reason relations do not. The perhaps paradoxical claim is that by widening our perspective to include *pre*logical reasoning practices of defending claims by giving *non*logical reasons for them and challenging claims by giving *non*logical reasons against them, we can bring into relief some of the fine structure of the "rootedness" of logical deduction in dialogical practices, which is Dutilh Novaes's focal concern. For that more encompassing perspective makes visible the role of *material* relations of rational implication and incompatibility in the elaboration of logical relations of deductive consequence and inconsistency, on the one hand, and the role of those reason relations in practices of rational dialogue, on the other.

One might worry that telling the story the way I have here stands in tension with one of the central contentions of the book. This is the observation that the practice of deductive logical proof is the product of quite specific, contingent, culture-bound, historically conditioned circumstances. Developing this thesis is one of the buttresses of Dutilh Novaes's account of the dialogical roots of deduction. I do not think the story I have sketched contradicts or even threatens the book's insight as to the rarified, historically situated character of the process by which deductive logical proofs crystallize out of stylized dialogic reasoning practices. Rather, I think the two perspectives are compatible and complementary. Engaging in logical deductive practices is indeed, as Dutilh Novaes teaches, a contingent, sophisticated, late-coming product of a quite specific tradition. But the logic that results from that tradition *expresses*, in its distinctive sophisticated, only contingently available way, fundamental features of reasoning in general—and so of discursive practices as such.

Appendix 1: A Toy Material Semantic Frame (MSF):

File - Dialogic Pragmatics_Main_August 7th

```
C:\Users\Bob\PycharmProjects\DP3\venv\Scripts\python.exe "C:/Users/Bob/Dropbox/NonMonCon/Spring 2019/Python/Dialogic Pragmatics_Main_August 7th.py"
You can retrieve this MSF using the Decode_MSF function with the fo11owing code:
len7imp360200272320539843346617167333742963878291733162880807977100934542987143753956722491414188559229879302574341596434451140864117500537480132854474110705384522311429722876902344780945169660327970716137626965987939030546649369528923116988172437 1
 95027309246090192871612782 0inc3639474140251240480754444105219686 46
This MSF contains in total 658 implications, among which 190 are pragmatically significant, 448 are required by CO, 7 are required by ExFF and 13 are strange in the sense that the premises and the conclusion are jointly persistently inconsistent.
 (Note that if an implication is required both by CO and ExFF, it's considered to be required by CO but not ExFF.)
This MSF contains 60 inconsistent sets, among which 6 are persistently inconsistent.
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
This MSF contains the following 190 pragmatically significant implications, i.e. implications that are not required by CO or ExFF and are not
strange.
'{0, 1, 2, 3, 4}|~5', '{0, 1, 2, 3, 5}|~6', '{0, 1, 2, 3}|~6', '{0, 1, 2, 5}|~4', '{0, 1, 2, 6}|~3',
'{0, 1, 2}|~4', '{0, 1, 2}|~5', '{0, 1, 3, 4, 5}|~2', '{0, 1, 3, 4, 5}|~6', '{0, 1, 3, 4, 6}|~ 5',
'{0, 1, 3, 4}|~2', '{0, 1, 3, 4} |~5', '{0, 1, 3, 5, 6}|~2', '{0, 1, 3, 5, 6} |~4', '{0, 1, 3, 5}|~2',
'{0, 1, 3, 5}|~4', '{0, 1, 3, 5}|~6', '{0, 1, 3, 6}|~2', '{0, 1, 3, 6}|~4', '{0, 1, 3, 6}|~5',
'{0, 1, 3}|~5', '{0, 1, 3}|~6', '{0, 1, 4, 5, 6}|~3', '{0, 1, 4, 5}|~3', '{0, 1, 4, 5}|~6',
'{0, 1, 4, 6}|~5', '{0, 1, 4}|~3', '{0, 1, 5, 6}|~3', '{0, 1, 5, 6}|~4', '{0, 1, 5}|~2',
'{0, 1, 5}|~3', '{0, 1, 5}|~4', '{0, 1, 5}|~ 6', '{0, 1, 6}|~2', '{0, 1, 6}|~3',
'{0, 1, 6}|~4', '{0, 1, 6}|~5', '{0, 1}|~2', '{0, 1} 4', '{0, 1}|~6',
'{0, 2, 3, 4}|~5', '{0, 2, 3, 4}|~6', '{0, 2, 3, 5, 6}|~1', '{0, 2, 3, 5}|~1', '{0, 2, 3, 5}|~4',
'{0, 2, 3, 5}|~6', '{0, 2, 3, 6}|~ 1', '{0, 2, 3}|~1', '{0, 2, 3}|~5', '{0, 2, 3}|~6',
'{0, 2, 4, 5}|~1', '{0, 2, 4, 6}|~ 3', '{0, 2, 4}|~5', '{0, 2, 4}|~6', '{0, 2, 5, 6}|~1',
 '{0, 2, 5, 6}|~3', '{0, 2, 5}|~6', '{0, 2, 6}|~1', '{0, 2, 6}|~4', '{0, 2, 6}|~5',
 '{0, 2}|~3', '{0, 2}|~4', '{0, 3, 4, 5}|~2', '{0, 3, 4, 6}|~1', '{0, 3, 4, 6}|~2',
'{0, 3, 4, 6}|~5', '{0, 3, 5, 6}|~1', '{0, 3, 5, 6}|~2', '{0, 3, 5, 6}|~4', '{0, 3}|~2',
'{0, 3}|~ 6', '{0, 4, 5, 6}|~1', '{0, 4, 5, 6}|~3', '{0, 4, 5}|~3', '{0, 4, 5}|~ 6',
'{0, 4, 6}|~ 2', '{0, 4, 6}|~ 3', '{0, 4}|~ 1', '{0, 4}|~ 2', '{0, 4}|~3',
'{0, 4}|~6', '{0, 5, 6}|~2', '{0, 6}|~1', '{0, 6}|~4', '{0, 6}|~5',
'{0}|~2', '{0}|~3', '{1, 2, 3, 4, 5}|~6', '{1, 2, 3, 4}|~ 6', '{1, 2, 3, 5, 6}|~ 0',
'{1, 2, 3, 6}|~0', '{1, 2, 3, 6} |~4', '{1, 2, 3, 6} |~5', '{1, 2, 3}|~6', '{1, 2, 4, 5, 6}|~ 3',
 '{1, 2, 4, 5}|~6', '{1, 2, 4, 6}|~ 5', '{1, 2, 4}|~3', '{1, 2, 5, 6}|~3', '{1, 2, 5}|~4',
'{1, 2}|~0', '{1, 2}|~ 6', '{1, 3, 4, 5, 6}|~2', '{1, 3, 4, 5}|~0', '{1, 3, 4, 5}|~6',
'{I, 3, 4, 6}|~0', '{1, 3, 4}|~0', '{I, 3, 4}|~5', '{I, 3, 5, 6}|~0', '{I, 3, 5, 6}|~4',
'{I, 3, 5}|~0', '{I, 3, 5}|~4', '{I, 3, 5}|~6', '{1, 3, 6}|~4', '{I, 3} |~0',
 '{1, 3}|~ 5', '{1, 3} |~ 6', '{1, 4, 6}|~ 3', '{1, 4}|~0', '{1, 4}|~6',
 '{1, 5, 6}|~ 4', '{1, 5}|~ 3', '{1, 5}|~ 4', '{1, 6}|~3', '{1, 6}|~5',
'{1}|~0', '{1}|~2', '{1}|~3', '{1}|~4', '{2, 3, 4, 5, 6}|~ 1',
'{2, 3, 4, 5}|~1', '{2, 3, 4, 6} |~', '{2, 3, 4, 6}|~ 5', '{2, 3, 4}|~1', '{2, 3, 4}|~5',
'{2, 3, 4}|~6', '{2, 3, 5, 6}|~1', '{2, 3, 5, 6}|~4', '{2, 3, 5}|~0', '{2, 3, 5}|~1',
'{2, 3, 5}|~4', '{2, 3, 5}|~6', '{2, 3, 6}|~0', '{2, 3, 6}|~1', '{2, 3, 6}|~5',
'{2, 3}|~4', '{2, 4, 5, 6}|~1', '{2, 4, 5}|~1', '{2, 4, 6}|~3', '{2, 4}|~1',
'{2, 4}|~6', '{2, 5, 6}|~0', '{2, 5, 6}|~3', '{2, 5}|~ 0', '{2, 5}|~3',
'{2, 5} |~4', '{2, 6}|~0', '{2, 6} |~3', '{2}|~3', '{2}|~4',
 '{2}|~ 5', '{2}|~6', '{3, 4, 5, 6} |~ 1', '{3, 4, 5}|~0', '{3, 4, 5}|~6',
 '{3, 4, 6}|~5', '{3, 4}|~1', '{3, 4}|~2', '{3, 5, 6} 4', '{3, 5}|~0',
'{3, 5}|~2', '{3, 5}|~4', '{3}|~2', '{4, 5, 6}|~1', '{4, 5, 6}|~2',
'{4, 5}|~3', '{4, 6}|~1', '{4, 6}|~3', '{4}|~0', '{4}|~1',
'{4}|~2', '{4}|~5', '{5, 6}|~1', '{5, 6}|~3', '{5}|~3',
'{5}|~6', '{6}|~0', '{6}|~3', '{6}|~4', '{6}|~ 5',
 '{5}'6', '{6}00', '{6}03', '{6}'4', '{6}05'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
This MSF contains the following 13 implications that are strange in the sense that the premises and the conclusion are jointly persistently inconsistent. We currently do not allow agents to use these implications as reason-fors.
 '{0, 1, 2, 3, 4, 5}|~ 6', '{0, 1, 2, 3, 4}|~ 6', '{0, 1, 2, 4}|~6', '{0, 1, 2, 5, 6}|~ 4', '{0, 1, 3, 4, 5, 6}|~2',
'{0, 1, 4, 5, 6}|~2', '{0, 2, 3, 4, 5}|~6', '{0, 2, 3, 5, 6} 4', '{|~, 2, 4, 6}|~1', '{0, 3, 4, 5, 6}|~2',
 '{I, 2, 3, 4, 5, 6}|~0', '{I, 2, 3, 4, 6}|~0', '{I, 2, 4, 5, 6}|~0'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^

Thus, this MSF has the following pragmatically significant reason-fors:
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
0 has the following 21 pragmatica11y significant reasons for itself:
'{1, 2, 3, 5, 6}', '{1, 2, 3, 6}', '{1, 2}', '{1, 3, 4, 5}', '{1, 3, 4, 6}',
 '{1, 3, 4}', '{1, 3, 5, 6}', '{1, 3, 5}', '{1, 3}', '{1, 4}',
 '{1}', '{2, 3, 4, 6}', '{2, 3, 5}', '{2, 3, 6}', '{2, 5, 6}',
'{2, 5}', '{2, 6}', '{3, 4, 5}', '{3, 5}', '{4}',
'{6}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
1 has the following 27 pragmatica11y significant reasons for itself:
'{0, 2, 3, 5, 6}', '{0, 2, 3, 5}', '{0, 2, 3, 6}', '{0, 2, 3}', '{0, 2, 4, 5}',
'{0, 2, 5, 6}', '{0, 2, 6}', '{0, 3, 4, 6}', '{0, 3, 5, 6}', '{0, 4, 5, 6}',
'{0, 4}', '{0, 6}', '{2, 3, 4, 5, 6}', '{2, 3, 4, 5}', '{2, 3, 4}',
'{2, 3, 5, 6}', '{2, 3, 5}', '{2, 3, 6}', '{2, 4, 5, 6}', '{2, 4, 5}',
 '{2, 4}', '{3, 4, 5, 6}', '{3, 4}', '{4, 5, 6}', '{4, 6}',
'{4}', '{5, 6}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
2 has the following 23 pragmatica11y significant reasons for itself:
'{0, 1, 3, 4, 5}', '{0, 1, 3, 4}', '{0, 1, 3, 5, 6}', '{0, 1, 3, 5}', '{0, 1, 3, 6}',
'{0, 1, 5}', '{0, 1, 6}', '{0, 1}', '{0, 3, 4, 5}', '{0, 3, 4, 6}',
'{0, 3, 5, 6}', '{0, 3}', '{0, 4, 6}', '{0, 4}', '{0, 5, 6}',
'{0}', '{1, 3, 4, 5, 6}', '{1}', '{3, 4}', '{3, 5}',
'{3}', '{4, 5, 6}', '{4}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
3 has the following 32 pragmatica11y significant reasons for itself:
'{0, 1, 2, 6}', '{0, 1, 4, 5, 6}', '{0, 1, 4, 5}', '{0, 1, 4}', '{0, 1, 5, 6}',
'{0, 1, 5}', '{0, 1, 6}', '{0, 2, 4, 6}', '{0, 2, 5, 6}', '{0, 2}',
'{0, 4, 5, 6}', '{0, 4, 5}', '{0, 4, 6}', '{0, 4}', '{0}',
'{I, 2, 4, 5, 6}', '{I, 2, 4}', '{I, 2, 5, 6}', '{I, 4, 6}', '{I, 5}',
'{I, 6}', '{1}', '{2, 4, 6}', '{2, 5, 6}', '{2, 5}',
'{2, 6}', '{2}', '{4, 5}', '{4, 6}', '{5, 6}',
 '{5}', '{6}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
4 has the following 30 pragmatica11y significant reasons for itself:
'{0, 1, 2, 5}', '{0, 1, 2}', '{0, 1, 3, 5, 6}', '{0, 1, 3, 5}', '{0, 1, 3, 6}',
'{0, 1, 5, 6}', '{0, 1, 5}', '{0, 1, 6}', '{0, 1}', '{0, 2, 3, 5}',
'{0, 2, 6}', '{0, 2}', '{0, 3, 5, 6}', '{0, 6}', '{I, 2, 3, 6}',
'{I, 2, 5}', '{I, 3, 5, 6}', '{I, 3, 5}', '{I, 3, 6}', '{I, 5, 6}',
'{1, 5}', '{1}', '{2, 3, 5, 6}', '{2, 3, 5}', '{2, 3}',
'{2, 5}', '{2}', '{3, 5, 6}', '{3, 5}', '{6}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
5 has the following 26 pragmatica11y significant reasons for itself:
{0, 1, 2, 3, 4} . {0, 1, 2} . {0, 1, 3, 4, 6} . {0, 1, 3, 4} . {0, 1, 3, 6} .
'{0, 1, 3}', '{0, 1, 4, 6}', '{0, 1, 6}', '{0, 2, 3, 4}', '{0, 2, 3}',
'{0, 2, 4}', '{0, 2, 6}', '{0, 3, 4, 6}', '{0, 6}', '{1, 2, 3, 6}',
'{I, 2, 4, 6}', '{I, 3, 4}', '{I, 3}', '{I, 6}', '{2, 3, 4, 6}',
'{2, 3, 4}', '{2, 3, 6}', '{2}', '{3, 4, 6}', '{4}',
'{6}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
6 has the following 31 pragmatically significant reasons for itself:
'{0, 1, 2, 3, 5}', '{0, 1, 2, 3}', '{0, 1, 3, 4, 5}', '{0, 1, 3, 5}', '{0, 1, 3}',
'{0, 1, 4, 5}', '{0, 1, 5}', '{0, 1}', '{0, 2, 3, 4}', '{0, 2, 3, 5}',
'{0, 2, 3}', '{0, 2, 4}', '{0, 2, 5}', '{0, 3}', '{0, 4, 5}',
'{0, 4}', '{1, 2, 3, 4, 5}', '{1, 2, 3, 4}', '{1, 2, 3}', '{1, 2, 4, 5}',
'{1, 2}', '{1, 3, 4, 5}', '{1, 3, 5}', '{1, 3}', '{1, 4}',
'{2, 3, 4}', '{2, 3, 5}', '{2, 4}', '{2}', '{3, 4, 5}',
'{5}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
This MSF contains the following 60 inconsistent sets:
'{0, 1, 2, 3, 4, 5, 6}', '{0, 1, 2, 3, 4, 6}', '{0, 1, 2, 3, 4}', '{0, 1, 2, 3, 6}', '{0, 1, 2, 4, 5, 6}',
'{0, 1, 2, 4, 6}', '{0, 1, 2, 4}', '{0, 1, 2, 6}', '{0, 1, 3, 4, 6}', '{0, 1, 3, 4}',
'{0, 1, 3, 5}', '{0, 1, 3, 6}', '{0, 1, 3}', '{0, 1, 4, 5}', '{0, 1, 4, 6}',
'{0, 1, 5, 6}', '{0, 2, 3, 4, 5, 6}', '{0, 2, 3, 4, 5}', '{0, 2, 3, 5, 6}', '{0, 2, 3, 5}',
'{0, 2, 4, 5, 6}', '{0, 2, 4, 5}', '{0, 2, 4, 6}', '{0, 2, 5, 6}', '{0, 2}',
'{0, 3, 4, 5, 6}', '{0, 3, 4, 6}', '{0, 3, 4}', '{0, 3, 5}', '{0, 3, 6}',
'{0, 4, 6}', '{0, 5}', '{I, 2, 3, 4, 5}', '{I, 2, 3, 4, 6}', '{I, 2, 3, 5, 6}',
'{I, 2, 3, 6}', '{I, 2, 3}', '{I, 2, 4}', '{I, 2, 5}', '{I, 2, 6}',
'{1, 3, 4, 5, 6}', '{1, 3, 4, 6}', '{1, 3, 4}', '{1, 3, 5}', '{1, 3}',
'{1, 4}', '{2, 3, 4, 5, 6}', '{2, 3, 4, 5}', '{2, 3}', '{2, 4, 6}',
'{2, 4}', '{2, 5}', '{3, 4, 5, 6}', '{3, 4, 6}', '{3, 4}',
'{3, 5, 6}', '{4, 5, 6}', '{4, 5}', '{4, 6}', '{5, 6}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
Among all inconsistent sets, the following 6 are persistently inconsistent:
['{0, 1, 2, 3, 4, 5, 6}', '{0, 1, 2, 3, 4, 6}', '{0, 1, 2, 4, 5, 6}', '{0, 1, 2, 4, 6}', '{0, 2, 3, 4, 5, 6}', '{0, 2, 4, 5, 6}']
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
Thus, this MSF contains the following reasons against:
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
0 has the following 32 reasons against itself:          {1, 2, 4, 5, 6}
{1, 2, 3, 4, 5, 6} . {1, 2, 3, 4, 6} . {1, 2, 3, 4} . {1, 2, 3, 6} .
'{1, 2, 4, 6}', '{1, 2, 4}', '{1, 2, 6}', '{1, 3, 4, 6}', '{1, 3, 4}',
'{1, 3, 5}', '{1, 3, 6}', '{1, 3}', '{1, 4, 5}', '{1, 4, 6}',
'{1, 5, 6}', '{2, 3, 4, 5, 6}', '{2, 3, 4, 5}', '{2, 3, 5, 6}', '{2, 3, 5}',
'{2, 4, 5, 6}', '{2, 4, 5}', '{2, 4, 6}', '{2, 5, 6}', '{2}',
'{3, 4, 5, 6}', '{3, 4, 6}', '{3, 4}', '{3, 5}', '{3, 6}',
'{4, 6}', '{5}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
1 has the following 28 reasons against itself:
'{0, 2, 3, 4, 6}', '{0, 2, 3, 4}', '{0, 2, 3, 6}', '{0, 2, 4, 6}', '{0, 2, 4}',
'{0, 2, 6}', '{0, 3, 4, 6}', '{0, 3, 4}', '{0, 3, 5}', '{0, 3, 6}',
 '{0, 3}', '{0, 4, 5}', '{0, 4, 6}', '{0, 5, 6}', '{2, 3, 4, 5}',
 '{2, 3, 4, 6}', '{2, 3, 5, 6}', '{2, 3, 6}', '{2, 3}', '{2, 4}',
'{2, 5}', '{2, 6}', '{3, 4, 5, 6}', '{3, 4, 6}', '{3, 4}',
'{3, 5}', '{3}', '{4}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
2 has the following 31 reasons against itself:
'{0, 1, 3, 4, 5, 6}', '{0, 1, 3, 4, 6}', '{0, 1, 3, 4}', '{0, 1, 3, 6}', '{0, 1, 4, 5, 6}',
'{0, 1, 4, 6}', '{0, 1, 4}', '{0, 1, 6}', '{0, 3, 4, 5, 6}', '{0, 3, 4, 5}',
'{0, 3, 5, 6}', '{0, 3, 5}', '{0, 4, 5, 6}', '{0, 4, 5}', '{0, 4, 6}',
'{0, 5, 6}', '{0}', '{1, 3, 4, 5}', '{I, 3, 4, 6}', '{1, 3, 5, 6}',
 '{I, 3, 6}', '{I, 3}', '{I, 4}', '{I, 5}', '{I, 6}',
 '{3, 4, 5, 6}', '{3, 4, 5}', '{3}', '{4, 6}', '{4}',
 '{5}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^

3 has the following 32 reasons against itself:
'{0, 1, 2, 4}', '{0, 1, 2, 6}', '{0, 1, 4, 6}', '{0, 1, 4}', '{0, 1, 5}',
'{0, 1, 6}', '{0, 1}', '{0, 2, 4, 5}', '{0, 2, 5, 6}', '{0, 2, 5}',
'{0, 4, 5, 6}', '{0, 4, 6}', '{0, 4}', '{0, 5}', '{0, 6}',
'{1, 2, 4, 5}', '{1, 2, 4, 6}', '{1, 2, 5, 6}', '{1, 2, 6}', '{1, 2}',
'{1, 4, 5, 6}', '{1, 4, 6}', '{1, 4}', '{1, 5}', '{1}',
'{2, 4, 5, 6}', '{2, 4, 5}', '{2}', '{4, 5, 6}', '{4, 6}',
'{4}', '{5, 6}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
4 has the following 36 reasons against itself:
'{0, 1, 2, 3, 5, 6}', '{0, 1, 2, 3, 6}', '{0, 1, 2, 3}', '{0, 1, 2, 5, 6}', '{0, 1, 2, 6}',
'{0, 1, 2}', '{0, 1, 3, 6}', '{0, 1, 3}', '{0, 1, 5}', '{0, 1, 6}',
'{0, 2, 3, 5, 6}', '{0, 2, 3, 5}', '{0, 2, 5, 6}', '{0, 2, 5}', '{0, 2, 6}',
 '{0, 3, 5, 6}', '{0, 3, 6}', '{0, 3}', '{0, 6}', '{1, 2, 3, 5}',
 '{1, 2, 3, 6}', '{1, 2}', '{1, 3, 5, 6}', '{1, 3, 6}', '{1, 3}',
'{1}', '{2, 3, 5, 6}', '{2, 3, 5}', '{2, 6}', '{2}',
'{3, 5, 6}', '{3, 6}', '{3}', '{5, 6}', '{5}',

^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
5 has the following 26 reasons against itself:
'{0, 1, 3}', '{0, 1, 4}', '{0, 1, 6}', '{0, 2, 3, 4, 6}', '{0, 2, 3, 4}',
'{0, 2, 3, 6}', '{0, 2, 3}', '{0, 2, 4, 6}', '{0, 2, 4}', '{0, 2, 6}',
 '{0, 3, 4, 6}', '{0, 3}', '{0}', '{1, 2, 3, 4}', '{1, 2, 3, 6}',
 '{I, 2}', '{I, 3, 4, 6}', '{I, 3}', '{2, 3, 4, 6}', '{2, 3, 4}',
'{2}', '{3, 4, 6}', '{3, 6}', '{4, 6}', '{4}',
 '{6}'
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
```

A Sample Dialogue

TurnNum	Agent	TargetNum	PragSig	Move	CL_AC	CL_RC	CL_AE	CL_RE	CR_AC	CR_RC	CR_AE	CR_RE
0	CL	None	proposal	['a_0', 'a_3', 'a_4', 'a_5'] entails a_2	[0, 2, 3, 4, 5]	[]	[0, 2, 3, 4, 5]	[	[]	[]	[]	[]
1	CR	0	premise challenge	['a_6'] excludes a_4	[0, 2, 3, 4, 5]	[]	[0, 3, 5]	[	[6]	[4]	[6]	[4]
2	CL	1	conclusion challenge	['a_0', 'a_1', 'a_3', 'a_5'] entails a_4	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[6]	[4]	[6]	[]
3	CR	2	conclusion challenge	['a_3', 'a_5', 'a_6'] excludes a_4	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 5]	[	[3, 5, 6]	[4]	[3, 5, 6]	[4]
4	CL	3	conclusion challenge	['a_1', 'a_3', 'a_5'] entails a_4	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[3, 5, 6]	[4]	[3, 5, 6]	[]
5	CR	0	conclusion challenge	['a_0'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[2]
6	CL	5	conclusion challenge	['a_1'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[]
7	CR	6	conclusion challenge	['a_5'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[2]
8	CL	7	conclusion challenge	['a_0', 'a_3'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[]
9	CR	8	conclusion challenge	['a_0', 'a_5', 'a_6'] excludes a 2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[2]
10	CL	9	conclusion challenge	['a_4'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 3, 5, 6]	[2, 4]	[0, 3, 5, 6]	[]
11	CR	10	conclusion challenge	['a_1', 'a_3', 'a_6'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
12	CL	11	conclusion challenge	['a_0', 'a_1', 'a_3', 'a_5'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
13	CR	12	conclusion challenge	['a_3'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
14	CL	13	conclusion challenge	['a_0', 'a_1'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
15	CR	14	conclusion challenge	['a_1', 'a_6'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
16	CL	15	conclusion challenge	['a_3', 'a_5'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
17	CR	16	conclusion challenge	['a_1', 'a_5'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
18	CL	17	conclusion challenge	['a_0', 'a_1', 'a_5'] entails a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 2, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
19	CR	18	conclusion challenge	['a_0', 'a_1', 'a_3', 'a_6'] excludes a_2	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 4, 5]	[	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
20	CL	19	premise challenge	['a_0', 'a_1', 'a_4'] excludes a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[]
21	CR	20	premise challenge	['a_0', 'a_1', 'a_5'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
22	CL	19	premise challenge	['a_0', 'a_1', 'a_5'] excludes a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[4]
23	CR	22	conclusion challenge	['a_0', 'a_3'] entails a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
24	CL	19	conclusion challenge	['a_0'] entails a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[4]
25	CR	24	conclusion challenge	['a_1', 'a_3', 'a_5', 'a_6'] excludes a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
26	CL	25	premise challenge	['a_0', 'a_1', 'a_3'] excludes a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[4]
27	CR	24	conclusion challenge	['a_1', 'a_3'] excludes a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2, 4]
28	CL	27	conclusion challenge	['a_3'] entails a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[4]
29	CR	28	conclusion challenge	['a_0', 'a_3', 'a_5'] excludes a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2, 4]
30	CL	21	conclusion challenge	['a_1'] entails a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2]
31	CR	26	conclusion challenge	['a_0', 'a_1', 'a_3'] entails a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 4, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2]
32	CL	29	conclusion challenge	['a_3', 'a_4'] entails a_2	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
33	CR	32	premise challenge	['a_0', 'a_3', 'a_6'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
34	CL	33	premise challenge	['a_5'] excludes a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[]
35	CR	34	conclusion challenge	['a_0', 'a_1'] entails a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
36	CL	33	conclusion challenge	['a_0', 'a_1', 'a_5'] entails a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
37	CR	36	conclusion challenge	['a_0', 'a_1', 'a_3'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
38	CL	37	conclusion challenge	['a_1', 'a_5'] entails a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[]
39	CR	38	conclusion challenge	['a_0', 'a_6'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5, 6]	[2, 4]
40	CL	39	premise challenge	['a_3', 'a_5'] excludes a_6	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[]
41	CR	38	conclusion challenge	['a_0', 'a_3'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2, 4]
42	CL	41	conclusion challenge	['a_3', 'a_5'] entails a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[]
43	CR	42	conclusion challenge	['a_1'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2, 4]
44	CL	43	conclusion challenge	['a_0', 'a_1'] entails a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[]
45	CR	44	conclusion challenge	['a_5'] excludes a_4	[0, 1, 2, 3, 4, 5]	[6]	[0, 1, 3, 5]	[6]	[0, 1, 3, 5, 6]	[2, 4]	[0, 1, 3, 5]	[2, 4]

```
By the end of this stage, next player has the following 0 for-moves available:

By the end of this stage, next player has the following 0 against-moves available:

By the end of this stage, CL's proposed conclusion is rejected.

Process finished with exit code 0
```

Reasoning, Reason Relations, and Semantic Content[1]

I. <u>Normative Pragmatics:</u> <u>From Reasoning Practices to Relations of Implication and Incompatibility</u>

A familiar order of explanation, inspired by Frege, begins with the distinction between two truth values, true and false. It then seeks to explain what is truth evaluable, what can have those truth values—what is expressed by what we can for that reason understand syntactically as declarative sentences—in terms of truth (or falsity) *conditions*. These are thought of as states of the world that in a distinctive semantic sense *make* the truth-evaluable sentences true or false. For that reason they are intelligible as what the sentences count as representing. In a suitable semantic metavocabulary, the truth conditions of, or truth-evaluable contents expressed by, declarative sentences take the mathematical form of functions from represented worldly states to the truth values of sentential representings of them.

The core task of pragmatics is offering an account of what one is doing in saying or thinking something expressible by the use of declarative sentences. In this semantics-first order of explanation, pragmatics is thought of as explanatorily downstream from the representational semantic story. The individual abilities exercised, or the social practices engaged in when speakers and thinkers use sentences with truth-*evaluable* contents or meanings are to be

[1] In this paper I deploy a number of arguments and report a number of results due to Ryan Simonelli (simonelli@uchicago.edu), Ulf Hlobil (ulf.hlobil@concordia.ca), and Dan Kaplan (dan.kaplan@pitt.edu), who are members of our logic working group "Research on Logical Expressivism." I mark their contributions as best I can along the way, to indicate what they are responsible for. They should not be presumed to endorse the use I have made of their work here.

understood in terms of truth-*evaluating* practical attitudes. Taking-true, in practice treating a sentence as expressing something true, is doxastically *accepting* it. Taking-false, in practice treating a sentence as expressing something false, is doxastically *rejecting* it. Those doxastic stances or practical attitudes can be manifested publicly by using sentences to perform speech acts of asserting and denying. It follows that what can be asserted or denied, doxastically accepted or rejected, is just what can take truth values.

A converse, pragmatics-first order of explanation begins with an account of the practical attitudes of doxastic acceptance and rejection, and seeks to understand in terms of them what is said or expressed by the declarative sentences used in the speech acts of assertion and denial that manifest those attitudes publicly. I think the best strategy for developing such a pragmatist semantics, a use-theory of meaning, do not lie in simply standing the traditional semantics-first story on its head. One can agree that doxastic acceptance can be characterized as taking-true and doxastic rejection as taking-false without treating those characterizations in terms of truth-evaluation as of use in substantial semantic explanations of what it is that interlocutors accept or reject: what is expressed by the declarative sentences they use to say something in the sense of asserting or denying it. Instead of understanding semantic content representationally, in terms of truth, one can look to further essential features of the discursive practices within which performances can have the pragmatic significance of assertions and denials, expressing doxastic practical attitudes of acceptance and rejection. For any autonomous discursive practice—any language game one can play though one plays no other—must include not only the making of claims, manifesting doxastic acceptance or rejection of them, but also practices of challenging and defending those claims by giving reasons for and against them. So another strategy would be to try to understand what is said or claimed, the contents that can be accepted or rejected doxastically, in terms of those reason relations among claimables: relations of being a reason for or against.

The claim is not that there cannot be acceptance and rejection in the absence of practices of giving reasons for and against the adoption of those attitudes. One might simply tick off one box rather than another, as on a ballot, menu, or scorecard. But such indications of preference for one or another option presuppose specifications of the contents of those options. The issue is what is required for that. The claim is that if one wants to understand *what* can be accepted or rejected, one should look to a fuller discursive context that includes practices of defending and challenging those attitudes by giving reasons for and against them.

The idea is to proceed in two stages in understanding semantics in terms of pragmatics, meaning in terms of use. The first stage begins with practices of reasoning: practices of defending and challenging commitments to accept and reject, undertaken in the first instance by performing speech acts of asserting and denying. On the basis of an account of such reasoning practices involving claim*ings*, it offers an account of the reason relations among claim*ables* (what can be doxastically accepted or rejected) in virtue of which some claim*ings* can serve as reasons for and against others. The second stage then elaborates a semantic understanding of the contents that can be doxastically accepted or rejected in terms of roles what is expressed by declarative sentences play in those reason relations. The core of such a two-phase, pragmatics-first semantic explanatory strategy is to use reason relations among claimable contents to mediate between a pragmatic account of what discursive practitioners *do* in making claims and giving and asking for reasons for them, on the one hand, and a semantic account of the claimable contents they assert and deny, defend and challenge by engaging in such practices, on the other.

Here is a sketch of how the first stage of such an account of the path from pragmatics to semantics might go. We can unpack the distinction and relation between practices of reasoning about claim*ings* (doxastic acceptances and rejections), and reason relations among claim*able* contents into these pieces:

1. Discursive practice as such involves reasoning because in addition to accepting and rejecting what is expressed by declarative sentences, interlocutors both *defend* and *challenge* the rational credentials of those stances or practical attitudes.
2. Defending (the credentials of) a claiming is producing further claimings that provide reasons *for* the acceptance or rejection being challenged. Challenging (the credentials of) a claiming is producing further claimings that provide reasons *against* the acceptance or rejection being challenged.
3. If accepting A functions practically as a reason *to accept* B, then A provides a reason *for* B, and if accepting A functions practically as a reason *to reject* B then A provides a

reason *against* B. Reason relations are relations that one set of claimables stands in to another when the first consists of reasons for or against the other.

4. We can call these reason relations "implication" and "incompatibility." To give a reason for is to commit oneself to accept premises that *imply* the claimable a reason is being given for. To give a reason against is to commit oneself to accept premises that are *incompatible* with the claimable a reason is being given against.[2]

To say symbolically that a set Γ of acceptables/rejectables *implies* acceptable/rejectable A, we can write "Γ|~A." Use of the "snake turnstile" rather than the more familiar double turnstile |= of semantic consequence or the single turnstile |– of derivability reminds us that we are expressing *rational* implications, not specifically *logical* implications. This is the sense in which "Pedro is a donkey" implies "Pedro is a mammal." Because the goodness of that implication depends on the contents of the nonlogical concepts <u>donkey</u> and <u>mammal</u>, rather than solely on the contents of logical concepts such as those expressed by conditionals and negation, Wilfrid Sellars calls these "materially" good implications. We can understand an implication as *logically* good in case it meets two conditions: i) it is materially good, and ii) it's material goodness is robust under arbitrary uniform substitution of nonlogical vocabulary for nonlogical vocabulary. We logical expressivists understand logical vocabulary as demarcated by a distinctive expressive role, whose paradigm is the way conditionals let us make explicit implications and negation lets us make explicit incompatibilities. But the story I am telling here addresses considerations that arise upstream of the introduction of specifically logical vocabulary to codify material reason relations.[3] For the other basic kind of reason relation, to say that a set Γ of acceptables/rejectables is *incompatible* with acceptable/rejectable A, we can write "Γ#A."

A promising direction in which such an account might be deepened and extended is suggested by Greg Restall and David Ripley's bilateralist normative pragmatic analysis of

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[2] These are the base cases that pragmatically define implication and incompatibility. On this basis more sophisticated practices can be built. One example would be giving a reason against a claim by rejecting some claimable that implies it.

[3] For a sketch of how this subsequent story goes, see my "From Logical Expressivism to Expressivist Logics" *Nous*: *Philosophical Issues,* Volume 28, Issue 1, October 2018, (a volume devoted to the philosophy of logic), pp. 70-88, reprinted in Ondrej Beran, Vojtech Kolman, Ladislav Koren (eds.) *From Rules to Meaning: New Essays on Inferentialism* (Routledge Studies in Contemporary Philosophy).

implication relations. They suggest that we understand the implication statement "$\Gamma|\sim A$" as saying that that the position in which one is committed to *accept* all of Γ and *reject* A is normatively out of bounds. This philosophically powerful pragmatic interpretation allows them to understand sequent calculi as consisting of rules that tell us that if some positions are out of bounds, then some others are also. (From their point of view, a principal benefit of the account is that it makes sense of multiple conclusion implications of the sort Gentzen introduces for classical logic. For we can say that Γ implies Δ just in case commitment to accept everything in Γ and reject everything in Δ is out of bounds. For the moment I ignore multisuccedent sequents, though they will become relevant to my story when I discuss the transition to semantics Part Two.) This bilateralist reading of implication understands the role of reason relations to be articulating norms that govern the adoption of practical doxastic attitudes of acceptance and rejection. They guide and constrain what interlocutors do by dividing constellations of attitudes into those that are appropriate and inappropriate (their "in bounds" and "out of bounds"), rather than by issuing imperatives that determine at any point what one must do.

The normative pragmatic role of reason relations of implication and incompatibility can be further elaborated by thinking about reasoning practices in terms of commitments and entitlements. Here the basic claim is that to be intelligible as practices of reasoning, in the sense of accepting and rejecting claimables and defending and challenging those stances with reasons for and against them, the participants in such practices must be understood as keeping track of two different sorts of normative status: the kind of *commitment* one undertakes or acknowledges in accepting or rejecting a claimable by asserting or denying a sentence expressing it, and the sort of *entitlement* to that status or practical attitude that is at issue when *reasons* are offered for or against it. Accepting or rejecting a claimable, paradigmatically by asserting or denying it, is taking a stand on it, adopting a stance towards it. It is committing oneself with respect to it, in the way one would by saying "Yea" or "Nay" to it in response to a suitable yes/no question.

What difference does it then make whether an interlocutor can offer reasons to accept what he has accepted or to reject what he has rejected? The *commitments* involved, the stances taken up, the attitudes adopted, are the same either way. But it is also an integral feature of specifically *doxastic* commitments that one's *entitlement* to those commitments is always potentially at issue.

For in taking up a doxastic stance one renders oneself liable to demands for justification, for exhibition of reasons to accept or reject the claim one has accepted or rejected. One may be challenged to show that the position one has adopted is normatively appropriate, "in bounds," one was entitled to adopt.

Reasons matter because other practitioners must distinguish between the acceptances and rejections the speaker in question is *entitled* to, in virtue of having *reasons* to adopt those attitudes, and those the speaker is *not* entitled to, because unable to defend those commitments by offering reasons when suitably challenged to do so. It follows that for each interlocutor there must be not only a difference between the attitudes (commitments) he has adopted and those he has not, but also, within those he has adopted, between those he is entitled to or justified in, has rational credentials for, and those that are *mere* commitments, bare of such accompanying entitlements. In *Making It Explicit* I argue that part of what turns practically on one's entitlement or justification—the second-person correlate of the first-person *responsibility* to defend one's commitments when one's entitlement is suitably challenged—is the testimonial *authority* of one's act: its capacity to license others to adopt a corresponding attitude. The essential point is that in addition to the *committive* dimension of assertional practice, there is the *critical* dimension: the aspect of the practice in which the rational propriety of those commitments, their justificatory status, is assessed. (The claim that the autonomous discursive practices in which some performances can have the significance of the undertaking of specifically *doxastic* commitments must include the in-principle liability of such commitments to challenges to their associated entitlements is entirely compatible with understanding such practices as built around a default-and-challenge structure, in which commitments count as in order until and unless suitably challenged by undertaking commitments that offer reasons against them.)

Restall-Ripley bilateralism explains implication in terms of a single pragmatic normative status: a constellation of acceptances and rejections being "in bounds," or, contrastingly, "out of bounds," appropriate or inappropriate, OK or not OK. Distinguishing the two normative statuses of commitment and entitlement and their contrasting statuses permits us to discern further fine structure. In these terms, to say that a constellation of acceptances and rejections is out of bounds is to say that it is a collection of commitments to which one cannot be jointly entitled. In terms of commitments and entitlements, we can lay alongside their analysis of implication the analysis of incompatibility I offer in *Making It Explicit*: two commitments are incompatible when commitment to one precludes entitlement to the other. Ryan Simonelli has nicely synthesized this understanding of incompatibility with the Restall-Ripley understanding of implication in the definitions:

5. Γ implies A (Γ|~A) just in case commitment to accept everything in the premise-set Γ precludes entitlement to *reject* A.
6. Γ is incompatible with (rules out) A (Γ#A) just in case commitment to accept everything in premise-set Γ precludes entitlement to *accept* A.

Like the original normative pragmatic bilateral account of implication, these principles make explicit what it is that practitioners need to *do* in order thereby practically to be taking or treating some claimables to imply or be incompatible with others. They need to take or treat some commitments as precluding entitlement to others, in keeping deontic score on their own and others' normative statuses.

This more articulated bilateral account of the normative pragmatic functional roles that relations among the acceptables/rejectables expressed by declarative sentences must play in order for them to count as *reason* relations of implication and incompatibility can be connected to the prior discussion of how such reason relations are intelligible as providing reasons for and against commitments in practices of defending and challenging them by the following principles.

7. Any set of commitments that *precludes entitlement* to *reject* A thereby *implicitly* commits one to *accept* A.
8. Any set of commitments that *precludes entitlement* to *accept* A thereby *implicitly* commits one to *reject* A.

We can think of these principles as codifying definitions of a concept of some commitments being *implicit* in others. In the case of implication, they are "implicit in" in the literal sense of "implied by" a premise-set. Here that fundamental, etymologically natural notion of implicitness is being extended to include reason relations of incompatibility, on the basis that the pragmatic definitions (5) and (6) of implication and incompatibility show them to be two species of one genus. On this account, a reason *against* a *rejection* is an implication with that conclusion, since Γ|~A says that commitment to all of Γ precludes entitlement to reject A. That is the same as a reason *for* an *acceptance*. Dually, an incompatibility Γ#A exhibits its premises Γ as providing both a reason *against* acceptance and (so) a reason *for* rejection.

Principles (1) through (8) outline an order of explanation that begins with a characterization of practices of making claims and defending and challenging them, and ends with a specification of the functional role relations among the contents that are accepted or rejected, defended and challenged must play in order properly to be understood as relations of implication and incompatibility among those claimables. This is the first step in the two-stage pragmatics-first strategy for understanding semantic content. The second step is then to show how to understand the semantic contents of the declarative sentences used to assert and deny in terms of reason relations of implication and incompatibility among those claimables. We turn next to that task.

[2640 words in large type.]

II. Semantics and Reason Relations

The idea to be pursued is that once we have an understanding from the side of pragmatics of the fundamental pair of opposite-but-complementary reason relations, implication and incompatibility, it will be possible to use them to formulate a semantic theory explicating the acceptables/rejectables expressed by declarative sentences. Understanding what can be doxastically accepted or rejected in terms of the roles declarative sentences can play in reason relations of implication and incompatibility would provide a purely pragmatic explication of a fundamental semantic concept: the concept of the *contents* expressed by those declarative sentences. What I want to do next is to explain two contributions to this enterprise that are made by recent work by two other members of the ROLE working group, Ulf Hlobil and Dan Kaplan. Hlobil offers an illuminating perspective on the relation between a pragmatic story along the lines I have been telling here and the best contemporary work in formal semantics. Kaplan shows in detail how a proper semantic account of the contents expressed by declarative sentences can be elaborated from the role those sentences play in reason relations of implication and incompatibility.

One of the most sophisticated, flexible, and expressively powerful formal semantic understandings of conceptual content available today is Kit Fine's truth-maker semantics.[4] It is built on a space of what he calls "states." We are invited to think of the states as facts or situations, but the notion is an adaptable one, sufficiently general to include whatever it is that we might think of as making declarative sentences true or false. A subset of the space of states is distinguished as the *possible* states. The only structure imposed on the state space is a partial ordering of part-hood: some states are parts of others. It is assumed that every subset of the space has a least upper bound. It can be thought of as the *fusion* of the elements of the subset: the unique whole of which they are all parts. The content or proposition expressed by a sentence

[4] Introduced in "A Theory of Truth-maker Content I: Conjunction, Disjunction, and Negation" *Journal of Philosophical Logic* (2017) 46:625-674.

A is then specified bilaterally, as a pair of sets of states: those "verifying" states that would make it true and those "falsifying" states that would make it false.

Like intensional semantics appealing to possible worlds, truth-maker semantics advances from the fundamental opposition of truth and falsity to a notion of content as truth conditions. It is more general in including also a notion of falsity conditions, which are not assumed in general to be uniformly computable from the truth conditions. Its basic notion of a <u>state</u> is more capacious than that of possible world. Possible worlds are included as special cases of states. For two states can be defined as *compatible* just in case their fusion is one of the states distinguished as *possible*. And a state can be understood as a possible world just in case it is a maximal possible state, in the sense of containing as parts every state compatible with it. Further flexibility (in the form of hyperintensionality) is secured by not restricting the state space to *possible* states, but embedding those in a larger structure that includes multiple distinct *im*possible ones. In addition, the mereological structure of the state space provides expressive resources in the truth-maker semantic metavocabulary that have no analogue in classical possible worlds semantics. The bilateral conception of content, including falsifiers as well as verifiers and not assuming that either sort of semantic interpretant can straightforwardly be computed from the other, turns out to pay large expressive dividends.

The truth-maker semantic framework permits various definitions of the reason relations of implication and incompatibility. As state *t* counts as incompatible with a set S of states just in case the fusion of it with all the states in S is an impossible state. We can then say that Γ # A just in case any fusion of verifiers of all the members of Γ with any verifier of A is an impossible state. On the side of implication, there are a number of different notions of semantic consequence definable in the truth-maker setting, and Fine considers it a signal virtue of his approach that it can express and compare such a variety of senses of "follows from." For instance, Γ verifier-entails A in case every state that verifies all the sentences of Γ verifies A.

Ulf Hlobil shows how the truth-maker framework allows the definition of a further notion of implication, which Fine does not consider.[5] We can say that

[5] Ulf Hlobil "The Laws of Thought and the Laws of Truth as Two Sides of One Coin" [ROLE: July 1, 2021]. [Update [ref.] as needed.]

9. Γ |~ Δ iff any fusion of a state that verifies all the members of Γ with a state that falsifies all the members of Δ is an impossible state.

He invites us to compare this semantic notion of multisuccedent implication with Restall and Ripley's bilateral pragmatic notion. Recall that they understand

10. Γ |~ Δ iff any position that includes accepting all of Γ and rejecting all of Δ is normatively incoherent or "out of bounds"—as we have read it: one cannot be entitled to such a constellation of commitments.

Both conceptions can be thought of as stemming from the same intuition that led C. I. Lewis to define his notion of <u>strict implication</u> by saying that in this sense of "implies" A implies B in case it is *impossible* for A to be true and B to be false. (It is the strengthening by necessitation of the horseshoe of bivalent classical logic.)

It is clear that these are isomorphic understandings of implication. The role played in the truth-maker semantic definition by verifiers and falsifiers of sentences is played in the bilateral pragmatic definition by practical attitudes of acceptance and rejection of sentences. And the role played in the truth-maker semantic definition by the impossibility of the state that results from fusing those verifiers and falsifiers is played in the bilateral pragmatic definition by the normative incoherence (or "out of bounds-ness") of the position that results from concomitant commitment to those acceptances and rejections. The isomorphism extends to incompatibility as well as implication. In the single-succedent formulation, we can lay alongside the truth-maker semantic reading:

11. Γ # A ⇔ the state resulting from *fusion* of any *verifiers* of all the members of Γ with any *verifier* of A is an *impossible* state,

the normative pragmatic reading:

12. Γ # A ⇔ the position resulting from *concomitant commitment* to *accept* all of Γ and to *accept* A is normatively *incoherent* ("out of bounds")—a constellation of commitments to which one *cannot* be entitled (entitlement is precluded).

I believe that this isomorphism between the definitions of reason relations of implication and incompatibility in the bilateral semantic framework of verifiers and falsifiers and the bilateral pragmatic framework of acceptance and rejection is deep and revealing. To begin with, it shows how the connection between two paired truth values and two paired doxastic attitudes expressed in the principles that accepting is taking-true and rejecting is taking-false is reflected, and can be further elaborated at the level of the reason relations of implication and incompatibility that articulate the contents that can *be* true/taken-true and false/taken-false. In particular, substantial new light is shed on what one must *do* to count thereby *as* adopting a practical attitude of taking some claimable to be true or false when those attitudes are situated in the wider context of practices of giving reasons for and against claimables that are constrained by reason relations of implication and incompatibility. The isomorphic relation between what is expressed by semantic metavocabularies of truth-makers and false-makers and what is expressed by pragmatic metavocabularies of bilateral commitments and preclusions of entitlement clarifies the relations between what one is *saying* and what one must be *doing* in order to say that in using the object language those semantic and pragmatic metavocabularies address. In practically acknowledging that commitment to accept some claimables precludes entitlement to reject some others and to accept still others, practitioners are, we can now see, *thereby* taking it that the fusion of verifiers of the premises and falsifiers (respectively, verifiers) of the conclusions are impossible states.

Alethic modal relations of possibility, impossibility, and necessity are part of the essential structure of the worldly states and situations that, according to the truth-maker semantic model, *make* claimables true or false, and so are what is represent*ed* and talked *of* or thought *about* by the use of declarative sentences. Deontic normative relations of commitment, entitlement, and preclusion of entitlement are part of the essential structure of discursive practical attitudes adoption of which, according to the pragmatics-first model, is what practitioners must *do* in order thereby to count as taking or treating what is expressed by declarative sentences *as* true or false, thereby represent*ing* the world as being some ways and not others by saying or thinking *that* things are thus-and-so. The very same reason relations of implication and incompatibility, which articulate the claimable contents expressed by declarative sentences, what can both *be* true or false and be practically *taken* to be true or false by affirming or denying them, can be construed *equally* and *isomorphically* both semantically, in alethic modal terms of *making* true or false, and pragmatically, in deontic normative terms of the practical doxastic attitudes of *taking* true or false (accepting or rejecting).

In *A Spirit of Trust* I attribute a view recognizably of this shape to Hegel, under the rubric "bimodal hylomorphic conceptual realism." He emphasizes reason relations of material incompatibility (Aristotelian contrariety) over those of implication or material consequence—his notion of "determinate negation" over his notion of "mediation"—though both are always in play. As I read him, Hegel begins with the thought that ways the world can objectively be, facts, are determinate just insofar as they exclude and entail one another in a way properly expressed in alethic modal terms. That the coin is copper makes it *impossible* that it remain solid at 1100 degrees Celsius and *necessitates* its being an electrical conductor. By contrast, our subjective takings of the world to be some way, thoughts, are determinate just insofar as they exclude and entail one another in a way properly expressed in deontic normative terms. As I've suggested here that we put this point, my commitment to the coin's being copper precludes *entitlement* to *accepting* that it would remain solid at 1100 degrees Celsius and precludes *entitlement* to *rejecting* that it is an electrical conductor. One and the same determinate conceptual content, that the coin is copper, can take two forms, an objective one in which it is understood as articulated by relations of exclusion and consequence construed in the alethic modal vocabulary proper to the expression of laws of nature, and a subjective one in which it is understood as articulated by relations of exclusion and consequence construed in the deontic normative vocabulary proper to the expression of discursive practices. That is why I use the term "bimodal hylomorphism." The view is properly denominated conceptual "realism" because the very same conceptual content to which we adopt attitudes in thought is understood as present, albeit in a different form, in the objective world thought about. The world is accordingly construed as essentially always already in a thinkable shape.

The isomorphism Hlobil has worked out between Restall and Ripley's normative pragmatic bilateral construal of implication and incompatibility relations and a version of Fine's truth-maker semantics is a colorable contemporary development of a thought cognate to the bimodal hylomorphic conceptual realism I attribute to Hegel. It suggests how something like this thought can be worked out in detail. For it maps onto one another a semantic idiom of great power and flexibility and a pragmatic idiom that has shown its substantial utility in understanding sequent calculi. Each has been used to characterize the fine structure of reason relations in actual applications to multifarious different object vocabularies.

When I introduced the idea of a pragmatics-first order of explanation, which would start with practices of accepting and rejecting and giving and asking for reasons entitling one to adopt those attitudes (so, challenging and defending doxastic commitments), I held out the prospect of a recognizably semantic understanding of the claimables that can be accepted or rejected (taken to be true or false) made available in terms of the reason relations of implication and incompatibility those claimables stand in to one another. We have seen how such reason relations can be understood in normative pragmatic terms of commitment and (preclusion of) entitlement, and how those very same reason relations can be reconstructed in paradigmatically semantic terms of worldly states or situations taken to make claimables true or false. But although the truth-maker semantics underwrites both a notion of the content expressed by declarative sentences and reason relations of implication and incompatibility that can also be understood in a normative pragmatic metavocabulary, it does not explain truth-evaluable content by appealing to those reason relations. Rather, it explains both in terms of modalized spaces of worldly states verifying and falsifying claimables. Striking as the isomorphism is that Hlobil points out between truth-maker semantic construals of implication and incompatibility and normative pragmatic construals of them, it does not amount to an explanation of claimable content *by means of* reason relations. So it does not by itself count as redeeming the promissory note I issued on behalf of a pragmatics-first order of semantic explanation.

To do that we can look to the implicational phase-space semantics (IPSS) developed by Dan Kaplan, a Pittsburgh member of our ROLE logic working group. It implements precisely what I have been promising: an understanding of what is expressed by declarative sentences in terms of the role those sentences play in reason relations of implication and incompatibility. In so doing it fulfills the defining aspiration of the philosophical tradition I call "semantic inferentialism." It begins with what I regard as a remarkable conceptual innovation. Not only are the semantic interpretants it appeals to implications (and incompatibilities), so is what is interpreted. That is, the principal and original bearers of semantic significance are construed not as sentences, but as implications.

The points of an implicational phase space are *candidate implications* defined on a language L_0 thought of as a set of logically atomic sentences. The candidate implications are then all ordered pairs $<\Gamma,\Delta>\in L_0 x L_0$ of sets of sentences of the language. They are what we have been representing by statements formed using the snake turnstile "$\Gamma|{\sim}\Delta$." This is the sort of thing manipulated in proof-theoretic multisuccedent sequent calculi—and given normative pragmatic interpretations by Restall-Ripley bilateralism. As is usual in such calculi, incompatibility is represented by empty right-hand sides rather than by a distinctive sort of turnstile: "$\Gamma,A|{\sim}$ " rather than "$\Gamma\#A$". (The empty right-hand side marks the incoherence of the set of premises that appears on the left-hand side of the turnstile.) I call the points of the implicational phase space "candidate" implications because they do not represent *good* implications: just candidates for that status. The good implications, the ones that actually hold—intuitively, where the set on the right-hand side, taken disjunctively, is a genuine consequence of the set of premises on the left-hand side, taken conjunctively, are marked as members of a distinguished subspace $\mathbf{I}_0$ of good implications.

The third element of an implicational phase-space semantic model for a language L_0—in addition to the space of candidate-implication points $L_0 x L_0$ and the subspace of good implications $\mathbf{I}_0$—is an operation $\dot{\cup}$ of *adjunction* of candidate implications. It is defined by:

<u>Adjunction</u>: $<\Gamma,\Delta> \dot{\cup} <\Phi,\Lambda> =_{df.} <\Gamma\cup\Phi, \Delta\cup\Lambda>$.

To adjoin two candidate implications one produces a third candidate implication by combining (in the sense of unioning) their premises and combining (in the sense of unioning) their conclusions. With the minimal candidate implication $<\emptyset,\emptyset>$ playing the role of an identity element, adjunction is a commutative monoid on the space $L_0 x L_0$.

Each candidate implication can now be assigned, as its semantic interpretant, the set of candidate implications whose adjunctions with it yield good implications, implications in the distinguished set $\mathbf{I}_0$.

<u>$\curlyvee$-sets</u>: $\forall x\in L_0 x L_0 \quad x^{\curlyvee} =_{df.} \{y\in L_0 x L_0 : x\dot{\cup}y\in\mathbf{I}_0\}$.[6]

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[6] $\curlyvee$-sets can be computed for *sets* of implications by requiring that each element of the $\curlyvee$-set yield an element of $\mathbf{I}$ when adjoined with *every* element of the set: $\forall X\subseteq L_0 x L_0\ X^{\curlyvee} =_{df.} \{ y\in L_0 x L_0 : \forall x\in X[x\dot{\cup}y\in\mathbf{I}_0]\}$.

The $\curlyvee$-set (pronounced “vee set”) of a candidate implication $\langle\Gamma,\Delta\rangle$ is what you need to add (adjoin) to it to get a *good* implication. If $\langle\Gamma,\Delta\rangle$ is already a good implication (if it is in $\mathbf{I}_0$) that fact will be marked by the fact that the minimal candidate implication $\langle\varnothing,\varnothing\rangle$ will be in its $\curlyvee$-set. If $\langle\Gamma,\Delta\rangle$ is a good implication, its $\curlyvee$-set $\langle\Gamma,\Delta\rangle^{\curlyvee}$ is something like its *range of subjunctive robustness*. Focusing for simplicity on the premise-set Γ, the $\curlyvee$-set is telling us what further collateral premises we can add to it without infirming the implication: turning it from a good one to a bad one. If the hungry lioness sees a limping gazelle nearby, then she will pursue it. That implication would still be good even if the beetle on a distant tree climbs a bit further out on the branch is it is sitting on. But it would not be good if the lioness were suddenly struck by lightning. If the candidate implication is not a good one, its $\curlyvee$-set tells us what we would need to add (adjoin) to it to *make* it a good one. Intuitively, the $\curlyvee$-sets play a role with respect to implications that is analogous to the role played by truth conditions with respect to sentences. They both specify what it would take for one to be semantically good—in the (different) ways implications and sentences can be semantically good.

At a second, separate stage, this semantic interpretation *of* (sets of) implications *by* sets of implications can then be extended to specify the semantic roles played by *sentences in* implications (and incompatibilities), rather than just of the implications themselves. In this implications-first inferentialist setting, a sentence A can be represented for semantic purposes by a pair of implications: $\langle \langle A,\varnothing\rangle, \langle\varnothing,A\rangle \rangle$. The semantic content expressed by the sentence—in the sense of its role in reason relations of implication and incompatibility—can then be represented by the $\curlyvee$-sets of these paired implications. $\langle A,\varnothing\rangle^{\curlyvee}$ determines the set of all the good implications in which A figures as a premise. $\langle\varnothing,A\rangle^{\curlyvee}$ determines the set of all the good implications in which A figures as a conclusion. For each tells us what additions to the bare skeletons of $\langle A,\varnothing\rangle$ and $\langle\varnothing,A\rangle$ yield good implications. The nature of the adjunction operation, in terms of which (together with the set of good implications $\mathbf{I}_0$) the $\curlyvee$-sets are defined, ensures that A appears as a premise in every element of the set of good implications that results from adjoining elements of $\langle A,\varnothing\rangle^{\curlyvee}$ to $\langle A,\varnothing\rangle$, and as a conclusion in every element of the set of good implications that results from adjoining elements of $\langle\varnothing,A\rangle^{\curlyvee}$ to $\langle\varnothing,A\rangle$.

The claim is that broadly inferential roles, in the sense specified by pairs of premissory and conclusory $\curlyvee$-sets $\langle\langle A,\emptyset\rangle^{\curlyvee}\ \langle\emptyset,A\rangle^{\curlyvee}\rangle$ are a good representation of what one must grasp in order to understand what one is accepting or rejecting in undertaking doxastic commitments.[7] For it is inferential roles in this sense that determine what is a reason for and against the claims to which one is committing oneself, and so what it would take to entitle oneself to those attitudes and the acts of affirmation and denial that overtly manifest them. For that reason, these are good semantic representations of the claimable contents expressed by declarative sentences. Of course the idea is not that in order to defend and challenge doxastic commitments we need to have fully mastered the intricacies of these inferential roles. It is that insofar as we do not, we do not know what we are committing ourselves to, do not fully understand what we are accepting or rejecting, or the reasons we give entitling us to do so.

A minimal criterion of adequacy for Kaplan's implicational phase-space semantics is that it can be shown to offer a tractable semantics for the logically complex sentences that result when we extend the logically atomic language L_0 by introducing sentential logical vocabulary according to a wide variety of sequent rules. Indeed, Kaplan proves soundness and completeness results using the implicational phase-space semantics for a number of such logics, including not only classical and intuitionistic logics, but also a wide variety of substructural (nonmonotonic, nontransitive, noncontractive…) logics). This broadly inferentialist semantic account of the claimable (acceptable/rejectable) contents expressed by declarative sentences is what I had in mind when I initially raised the possibility that a pragmatics-first approach that understands reason relations of implication and incompatibility in normative terms of what one is *doing* in adopting doxastic practical attitudes of accepting and rejecting claims and challenging and defending entitlement to the resulting commitments by offering reasons for and against them could be built on, extended, and developed to provide an adequate semantics.

I have gestured at two routes to semantics. I have described how Hlobil offers a way of understanding (his version of) reason relations, paradigmatically implication, in Fine's truth-maker semantics, in terms of an isomorphism with Restall and Ripley's bilateralist normative

[7] For both conceptual and technical reasons, it turns out that it is best to use the closures of these $\curlyvee$-sets under the $\curlyvee$-function, which can be shown to reach a fixed point at $\langle\langle A,\emptyset\rangle^{\curlyvee\curlyvee\curlyvee}\ \langle\emptyset,A\rangle^{\curlyvee\curlyvee\curlyvee}\rangle$, but I suppress this complication.

pragmatics. I have explained in general terms how Kaplan defines his implicational phase-space semantics directly in terms of implication (and incompatibility) relations, which we have seen can be understood in normative pragmatic terms of acceptance and rejection, commitments and (preclusions of) entitlement. I want to close this section by comparing and contrasting the reconstructions of reason relations of implication and incompatibility offered by these two semantic approaches: in terms of truth-makers and in terms of implications.

The first thing to appreciate is the strong formal analogies between the two frameworks. The modalized state spaces of truth-maker semantics are built on sets of "states" that are not further specified. The states making up these spaces could be pretty much anything—which of course contributes greatly to the flexibility of the apparatus. Within the set S of states, a privileged subset of "good" ones, $S^{\Diamond}$ is distinguished—intuitively, by its alethic modal status as "possible." Kaplan's implicational phase spaces are sets of points that have more structure than Fine's states. They are candidate implications, pairs of sets of sentences drawn from an antecedent prelogical language. Within this space $L_0 x L_0$ of implications, a privileged subset of "good" ones, $\mathbf{I}_0$ is distinguished—intuitively, by its normative status as codifying the proper implications, what really follows from what. The operations on states or candidate implications, fusion $\sqcup$ and adjunction $\uplus$ (the one stipulated, the other defined in terms of the additional structure of the space of candidate implications defined on L_0) are algebraically both commutative monoids.[8] The semantic interpretants of sentences are in both cases bilateral: verifiers/falsifiers and premissory and conclusory $\curlyvee$-sets respectively.

There is also a substantial formal difference between the two settings. In the truth-maker framework, the modalized state space with its fusion operation (or part-whole relation among states) is wholly distinct from the language it is used to interpret semantically. To get a semantic model, a third element is required: an interpretation function that maps sentences of the language onto pairs of sets of verifying and falsifying states. In the implicational phase-space framework, there is nothing corresponding to this extra element, connecting and mediating between the

[8] Both Fine's truth-maker semantics on modalized state spaces and Kaplan's implicational phase-space semantics use commutative monoids (the fusion/adjunction operation, together with a null space unit element) on spaces with distinguished subspaces ($S^{\Diamond}$ and $\mathbf{I}$). This is an algebraic generalization of more familiar residuated lattices. In making this generalization, both are downstream from Girard's phase-space semantics for linear logic.

language and the space on which it is interpreted. The extra structure that the points of the implicational phase-space come with, their being candidate implications in the form of pairs of sets of sentences of the language, not only means that the monoidal operation of adjunction of candidate implications can be explicitly defined set-theoretically, as opposed to simply stipulated, as with fusion of states. Because the sentences themselves are already present in the space from which semantic interpretants are drawn, the $\curlyvee$-function that semantically interprets first implications and then sentences can also be explicitly defined set-theoretically from the raw materials already present in the implicational phase-space itself. In this sense, the interpretation function connecting sentences to their semantic intepretants is *intrinsic* to the sentences as they figure in the space of implications. The sentences come already interpreted by the reason relations they stand in to one another, the roles they play in implications and incompatibilities. All the semantic framework does is draw that intrinsic interpretation out explicitly. Now whether this is a virtue or a vice, a benefit or a cost, will depend on collateral theoretical commitments. For one might see it as showing that the implicational phase-space framework is foolishly trying to do without relations to extralinguistic reality that are what make truth-maker semantics a genuine *semantics* in the first place. I am not going to argue about that. But I do want to assemble some further considerations that might bear on such a dispute.

For in spite of the substantial difference in the conceptions of semantic interpretation that animate the two different approaches, the fact that both take the mathematical form of commutative monoids plus distinguished subspaces means that their treatment of the crucial reason relations of implication and incompatibility share enough structure to be intertranslatable across the two settings. That is, we can specify exactly the same reason relations of implication and incompatibility while moving systematically between the modalized state spaces of truth-maker semantics and implicational phase-space semantics. Here's how.

For one direction: Beginning with a truth-maker model, one can define an implicational phase space that corresponds to it in the sense of defining exactly the same implications and incompatibilities. We are given a truth-maker model of a language L_0, defined on a modalized state space $\langle S,S^{\Diamond},\sqcup\rangle$, which assigns to each sentence $A \in L_0$ a pair of sets of states $\langle v(A),f(A)\rangle$ understood as verifiers and falsifiers of that sentence. The points of the implicational phase space being defined are ordered pairs of sets of sentences of L_0. These are the candidate implications. What corresponds to fusion, $\sqcup$, is adjunction: $\langle\Gamma,\Delta\rangle \mathbin{\dot\cup} \langle\Theta,\Psi\rangle = \langle\Gamma\cup\Theta,\Delta\cup\Psi\rangle$, as usually defined in implicational phase space semantics. It remains to compute $\mathbf{I}_0$, the set of *good* implications. We do that using the consequence relation Hlobil defined to mimic the Restall-Ripley bilateral understanding of the multisuccedent turnstile:

$$\langle\Gamma,\Delta\rangle\in\mathbf{I}_0 \quad \text{iff} \quad \forall s,t\in S[\ (\forall G\in\Gamma[s\in v(G)]\ \&\ \forall D\in\Delta\ [t\in f(D)]\ ) \Rightarrow s\sqcup t\notin S^{\Diamond}\ ].$$

That is, $\langle\Gamma,\Delta\rangle$ is a good implication just in case the fusion of any state s that verifies all of Γ and any state t that falsifies all of Δ is an impossible state, in the truth-maker model. This construction obviously guarantees that exactly the same implications will hold in the implicational phase space, that is, be elements of $\mathbf{I}_0$, as satisfy the Hlobil consequence relation in the truth-maker model.

As for incompatibilities, in the truth-maker setting, two *states* s and t are incompatible just in case their fusion is an impossible state. Two *sentences* A and B are incompatible just in case any fusion of a verifier of the one with a verifier of the other is an impossible state. More generally, a set Γ of sentences is *incoherent* in case any fusion of verifiers of all its elements is an impossible state. Given the definition of the set of good implications $\mathbf{I}_0$ just offered, this is equivalent to $\langle\Gamma,\varnothing\rangle\in\mathbf{I}_0$. The incompatibilities are represented in the implicational phase space semantics just by good implications with empty right-hand sides.

So there is a straightforward method for taking any truth-maker model defined on a modalized state space and defining from it an implicational phase space model that has exactly the same reason relations of implication and incompatibility.

For the other direction: Beginning with an implicational phase space, one can define a truth-maker model (an interpreted modalized state space) that corresponds to it in the sense of defining exactly the same implications and incompatibilities. We are given an implicational phase space defined on a language L_0, $\langle\mathcal{P}(L_0) \times \mathcal{P}(L_0), \mathbf{I}_0\rangle$. The states will be candidate implications. $S = \mathcal{P}(L_0) \times \mathcal{P}(L_0)$. $\sqcup$ is adjunction: $\langle\Gamma,\Delta\rangle \sqcup \langle\Theta,\Psi\rangle = \langle\Gamma\cup\Theta,\Delta\cup\Psi\rangle$. In the Hlobil truth-maker definition of consequence, the *good* implications correspond to *im*possible states. So the subset of *possible* states is defined by $S^{\Diamond} = S\text{-}\mathbf{I}_0$. It remains to define the model function m, which assigns to each $A\in L_0$ a pair of subsets of S, $\langle v(A),f(A)\rangle$, where $v(A)\subseteq L_0$ and $f(A)\subseteq L_0$, such that:

$\langle\Gamma,\Delta\rangle \in \mathbf{I}_0$ iff $\forall s,t \in S[(\Gamma=\{G_1\ldots G_n\}$ & $g_1 \in v(G_1)$ &…$g_n \in v(G_n)$ & $s=g_1\sqcup\ldots\sqcup g_n$ & $\Delta=\{D_1\ldots D_n\}$ & $d_1 \in v(D_1)$ &…$d_n \in v(D_n)$ & $t=d_1\sqcup\ldots\sqcup d_n\ ) \Rightarrow s\sqcup t \notin S^{\Diamond}\ ]$.

For various metatheoretic purposes, Fine employs "canonical" truth-making models, in which the verifier of a (logically atomic) sentence is just that sentence and the falsifier of that sentence is just the negation of that sentence. (His requirement that the fusion of any verifiers of A will be a verifier of A and the fusion of any falsifiers of A will also be a falsifier of A is then trivially satisfied, since there is only one.) We can combine that idea with Kaplan's standard representation of the proposition expressed by A as the pair $\langle \langle A, \varnothing\rangle, \langle\varnothing, A\rangle \rangle$, and do without the formation of falsifying literals by appeal to negation by defining the verifiers of A by $v(A) = \langle A, \varnothing\rangle$ and the falsifiers of A by $f(A) = \langle\varnothing, A\rangle$. We want to implement Hlobil's definition of implication (generalizing C. I. Lewis's strict implication to Fine's truthmaker semantic framework), that an implication $\Gamma|\!\sim\Delta$ is good in the truth-maker setting just in case the fusion of any verifier of all of Γ and any falsifier of all of Δ is an impossible state. To do that, we need to say what it is for a state (defined in the implicational phase space, that is, a candidate implication) to "verify all of Γ" and to "falsify all of Δ." We can extend the single-sentence definitions as follows. If $\Gamma=\{G_1\ldots G_n\}$ and $\Delta=\{D_1\ldots D_m\}$:

$$v(\Gamma) = \langle\Gamma,\varnothing\rangle = \langle G_1,\varnothing\rangle \dot{\cup}\ldots\dot{\cup} \langle G_n,\varnothing\rangle.$$

$$f(\Delta) = \langle\varnothing,\Delta\rangle = \langle\varnothing,D_1\rangle \dot{\cup}\ldots\dot{\cup} \langle\varnothing,D_m\rangle.$$

That is, the implication (standing in for a state) $\langle\Gamma,\varnothing\rangle$ counts as verifying all of Γ because it is the adjunction of the verifiers of each element of Γ. (In this "canonical" modalized state-space model, sets of sentences, like individual sentences, only have single states=implications as verifiers.) And similarly for falsifiers.

To show that this works, in the sense of yielding the same implications in the truth-maker model that are good in the original implicational phase space, we must show that

$$\langle\Gamma,\Delta\rangle \in \mathbf{I}_0 \text{ iff } \forall s,t \in S[\ (\forall G\in\Gamma[s\in v(G)] \ \&\ \forall D\in\Delta\ [t\in f(D)]\ ) \Rightarrow s\sqcup t\notin S^{\Diamond}\ ].$$

<u>To show the left-to-right direction $\Rightarrow$</u>: If $\langle\Gamma,\Delta\rangle \in \mathbf{I}_0$ then $v(\Gamma)=\langle\Gamma,\varnothing\rangle$ and $f(\Delta)=\langle\varnothing,\Delta\rangle$. So $v(\Gamma)\sqcup f(\Delta)=\langle\Gamma,\Delta\rangle$. Since by hypothesis $\langle\Gamma,\Delta\rangle \in \mathbf{I}_0$, by the definition of $S^{\Diamond}$ as $S\text{-}\mathbf{I}_0$, it follows that $\langle\Gamma,\Delta\rangle \notin S^{\Diamond}$, that is, that the state $\langle\Gamma,\Delta\rangle$ is an impossible state. It is the fusion of *the* verifier of Γ, $\langle\Gamma,\varnothing\rangle$ and *the* falsifier of Δ $\langle\varnothing,\Delta\rangle$ because it is the result of adjoining them.

To show the right-to-left direction ⇐: If $\forall s,t\in S[(\Gamma=\{G_1\ldots G_n\}$ & $g_1\in v(G_1)$ &…$g_n\in v(G_n)$ & $s=g_1\sqcup\ldots\sqcup g_n$ & $\Delta=\{D_1\ldots D_n\}$ & $d_1\in v(D_1)$ &…$d_n\in v(D_n)$ & $t=d_1\sqcup\ldots\sqcup d_n) \Rightarrow s\sqcup t\notin S^\Diamond]$, then $s = v(\Gamma)$ and $t = f(\Delta)$, so $v(\Gamma)\sqcup f(\Delta)=<\Gamma,\Delta> \notin S^\Diamond$. Since $S^\Diamond = S-\mathbf{I}_0$ and $<\Gamma,\Delta>\in S$, $<\Gamma,\Delta>\in\mathbf{I}_0$.

As for incompatibility, we must show that A and B are truth-maker incompatible (Γis truth-maker incoherent), that is, $\forall s,t\in S[s\in v(A)$ & $t\in v(B) \Rightarrow s\sqcup t\notin S^\Diamond]$, (or more generally, $v(\Gamma)\notin S^\Diamond$) iff $<\{A,B\},\varnothing>\in\mathbf{I}_0$ (or more generally, $<\Gamma,\varnothing>\in\mathbf{I}_0$).

To show the left-to-right direction ⇒: If $\forall s,t\in S[s\in v(A)$ & $t\in v(B) \Rightarrow s\sqcup t\notin S^\Diamond]$, then since $v(A) = <A,\varnothing>$ and $v(B) = <B,\varnothing>$, and since ⊔ is adjunction, $s\sqcup t = <\{A\}\cup\{B\},\varnothing> = <\{A,B\},\varnothing>$. Since $\Rightarrow s\sqcup t\notin S^\Diamond$, $s\sqcup t = <\{A,B\},\varnothing>\in\mathbf{I}_0$. This works for arbitrary iterations of ⊔, which gives the more general Γcase.

To show the right-to-left direction ⇐: If $<\{A,B\},\varnothing>\in\mathbf{I}_0$, then $<\{A\}\cup\{B\},\varnothing>\in\mathbf{I}_0$.

Since ⊔ is adjunction, $<A,\varnothing>\sqcup<B,\varnothing>\in\mathbf{I}_0$. But $v(A) = <A,\varnothing>$ and $v(B) =<B,\varnothing>$.

So $v(A)\sqcup v(B)\in\mathbf{I}_0$. Since $S^\Diamond = S-\mathbf{I}_0$, $v(A)\sqcup v(B)\notin S^\Diamond$. That is truth-maker incompatibility of A and B. This works for arbitrary iterations of ⊔, which gives the more general Γ case.

So there is a straightforward uniform translation between Kaplan's implicational phase-state semantics and Fine's truth-maker semantics. Each truth-maker model on a language corresponds to an implicational phase-space defined on that same language, in the sense that they underwrite exactly the same reason relations of implication and incompatibility. The parallel extends to various structural constraints that can be placed on them—Fine's Exclusivity, Downward Closure, and Exhaustivity conditions, which I'll have more to say about further along.

This translation shows how truthmaker semantics can be "deflated" from the point of view of semantic inferentialism. For it shows how to extract what the inferentialist insists is its semantic core: the way it functions to codify reason the relations of implication and incompatibility that articulate claimable (so, propositional) contents. The representational, metaphysical reading of "truthmaking states" is, from this perspective, optional and inessential: at best a harmless indulgence, at worst a misleading characterization of the semantic enterprise. The position that results is the extension to the more sophisticated truthmaking and implicational phase-space semantics of the inferentialists views about classical model-theory and proof-theory. Both are seen as providing metavocabularies for codifying reason relations of implication and incompatibility. In the classical case, the differences in the expressive power of representational and inferential metavocabularies is interesting and instructive, but not a reason to see one or the other approach as simply wrong-headed. The isomorphism between truthmaking and implicational phase-space semantics (the latter accompanied by, and sound and complete with respect to, a powerful sequent calculus) should engender the same irenic attitude toward these semantic metavocabularies.[9]

[5,144 words in large type.]

Conclusion

I have sketched an order of explanation that moves from pragmatics to semantics. The most basic use of linguistic expressions is to perform speech acts of assertion and denial, manifesting doxastic attitudes of accepting and rejecting. I claimed that what makes the practical attitudes *doxastic* attitudes (and so makes the locutions that express them *declarative sentences*) is their standing liability to *challenges* by assertions that provide reasons *against* them, and the consequent obligation to *defend* them by assertions that provide reasons *for* them. Those dialogic practices make intelligible reason relations of implication and incompatibility, which can be understood in terms of normative statuses of *commitment* to accept and reject and (preclusion of) *entitlement* to such commitments. The second stage of the envisaged pragmatics-first order of explanation then semantically characterizes the claimable contents expressed by the declarative sentences that are asserted or denied, what can be doxastically accepted or rejected,

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[9] I have been talking about how the "internal" consequence (and incompatibility) relations line up in the two settings. Looking somewhat further afield, the deep affinities between these two semantic approaches are also manifested in the way verifiers line up with premissory roles and falsifiers with conclusory roles, in the *external* consequence relations. (The internal relations cross the turnstile(s). The external ones remain on one side of the turnstile, looking at relations between the premissory sides of different sequents, or between the conclusory sides of different sequents. In substructural cases, the internal and external consequence relations can diverge.) Kaplan shows that K3 (the Strong Kleene three-valued logic) is the unilateral external logic of premissory roles in codifying the sense of consequence in which $A|=_pB$ just in case if in the internal logic $\Gamma,B|\sim\Delta$ then $\Gamma,A|\sim\Delta$ (A can replace B as a premise, saving the goodness of implications), and LP (Graham Priest's "Logic of Paradox") is the unilateral external logic of conclusory roles in codifying the sense of consequence in which $A|=_cB$ just in case if in the internal logic $\Gamma|\sim A, \Delta$ then $\Gamma|\sim B, \Delta$ (B can replace A as a conclusion, saving the goodness of implications). Hlobil shows that K3 is the unilateral external "logic of verifiers," in the sense that K3 preserves compatibility with the verifiers of the premises (jointly) to the verifiers of the conclusions (separately). And Hlobil shows that LP is the unilateral "logic of falsifiers," in the sense that LP preserves the compatibility potential of the falsifiers of the conclusions (jointly) to the falsifiers of the premises (separately). So the isomorphism between the reason relations specified by the truth-maker semantics and those specified by the implicational phase-space semantics goes beyond the internal (bilateral) consequence relations all the way to the external (unilateral) consequence relations as well.

in terms of the functional roles those sentences play in reason relations of implication and incompatibility. Dan Kaplan's substructural implicational phase-space semantics shows in detail how an expressively powerful formal semantics can be elaborated from the material relations of implication and incompatibility that precipitate out of the functionalist story told in such a normative pragmatic metavocabulary. This is the principal story I want to put on the table.

An exciting recent result of Ulf Hlobil's shows that Kit Fine's truthmaker semantics stands in surprising relations to the normative pragmatics gestured at here, and I build on that result to show that it also stands in surprising relations to Kaplan's inferentialist semantics. As a secondary project, I have sketched how those results can be used to facilitate the comparison of the pragmatics-first order of explanation with the most sophisticated contemporary development of the semantics-first order of explanation.

For we see first that the pragmatic significance of relations of implication and incompatibility defined in truthmaker terms can be articulated in bilateralist terms of the Restall-Ripley sort, and, by extension, in terms of normative statuses of commitment and entitlement suggested here to sharpen their account. This new way of building a pragmatics on top of truthmaker semantics marks a fault line, or at least a division of labor, within the truthmaker setting. On one side, there is the metaphysical story about states, about their mereological fusion, and about the division of them into possible and impossible. On the other side there are the reason relations of implication and incompatibility that Hlobil shows how to define on that basis. The pragmatic connection to discursive practices of defending and challenging doxastic attitudes of acceptance and rejection depends *only* on the latter. This is the only part of the semantic story that shows up as pragmatically significant. To show that the metaphysics of matters to (or, further, is even implicit in) the *use* of the expressions to whose meaning it purports to contribute, some further pragmatic story will have to be told, going beyond the one envisaged here.

Further, we saw that the meaning relations of implication and incompatibility generated by the heavily metaphysically committive truthmaker semantics can be reproduced exactly within the much less metaphysically committive implicational phase-space semantics, all of whose primitives can be understood as implicit in the pragmatics that specifies the use of expressions in discursive practice. So, space is opened for a *deflated* version of the truthmaker semantics. The claim here would be that the commutative monoid that does the heavy lifting in truthmaker semantics has been mischaracterized, needlessly and misleadingly encrusted with functionally irrelevant baroque metaphysical ornamentation. Underneath that misleading guise, what is doing the work is Kaplan's operator adjoining premises and conclusions of implications to mark their ranges of subjunctive robustness.[10]

Let me close with the observation that the very same normative pragmatic metavocabulary—of commitments to accept or to reject, and of preclusion of entitlement to such commitments—that can be used to specify the reasoning practices in which sentences of the prelogical object language are used to make, challenge, and defend claims, *also* suffices to specify the use of truth-first semantic metavocabularies (including the Fine's hyperintensional truthmaker version) to characterize both reason relations of implication and incompatibility and what is expressed by the declarative sentences that can be accepted or rejected, true or false. For the pragmatic metavocabulary for the truth-first semantic metavocabulary underlines the fact that what the semantic theorist is *doing* in sorting or evaluating claimables to begin with as true or false (perhaps guided by a view about what states would verify or falsify them) is just what the pragmatic metavocabulary takes as adopting the basic practical doxastic attitudes: *taking*-true (accepting) and *taking*-false (rejecting). The pragmatics-first order of explanation begins by explicitly theorizing about those practical attitudes as they show up in the use of the object language. The semantics-first order of explanation begins by practically adopting such attitudes, implicitly, and in an untheorized way, as part of the unexplained, taken-for-granted use of its semantic metavocabulary. The attitudes are fundamental in either case. The difference is just how theoretically and methodologically self-conscious one is about them. In the semantics-first order of explanation, the issue of what one is doing in making truth evaluations in the semantic metavocabulary, and in particular, what reasons entitle one to privilege *these* takings-true and

[10] It should be acknowledged that the isomorphism with Kaplan's implicational phase-space semantics has been shown to hold only when consequence in the truthmaker setting is defined the way Hlobil does in order to map that semantic setting onto Restall-Ripley bilateralist normative pragmatics. This is not how Fine himself defines consequence. He considers and employs a variety of such definitions, but taking an implication to be good if and only if the fusion of truthmakers of all the elements of the premise set and falsemakers of all the elements of the conclusion set is an impossible state, though natural enough, is not one of them.

takings-false (acceptances and rejections) is resolutely kept off-stage. This seems a point in favor of the pragmatics-first approach.

End